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Color-Based Broadcasting for Ad Hoc Networks

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***Abstract*— This paper develops a novel color-based broadcast scheme for wireless ad hoc networks where each forwarding of the broadcast message is assigned a color from a given pool of colors. A node only forwards the message if it can assign it a color from the pool which it has not already overheard after a random time. In the closely related counter-based broadcast scheme a node simply counts the *number* of broadcasts not the *colors* overheard. The forwarding nodes form a so-called backbone, which is determined by the random timers and, thus, is random itself. Notably, any counter-generated backbone could result from pruning a color-generated backbone; the typical color-generated backbone, however, exhibits a connectivity graph richer than the counter-based ones. As a particular advantage, the colors reveal simple geometric properties of the backbones which we exploit to prove that the size of both, color- and counter-generated backbones are within a small constant factor of the optimum. We also propose two techniques, *boosting* and *edge-growing*, that improve the performance of color- and counter-based broadcast in terms of reachability and number of rebroadcasts. Experiments reveal that the powerful boosting method is considerably more effective with the color-based schemes.**

I. MOTIVATION AND BACKGROUND

In ad hoc networks, broadcast plays a crucial role, relaying a message generated by one node to all other nodes. Several unicast routing protocols such Dynamic Source Routing (DSR), Ad Hoc On Demand Distance Vector (AODV), Zone Routing Protocol (ZRP), and Location Aided Routing (LAR), as well as multicast protocols employ broadcasting to detect and maintain routes in an ever changing environment.

The simplest approach for broadcasting is *flooding*, where each node rebroadcasts a message as soon as it receives it for the first time. While ensuring a high success rate in reaching all nodes, flooding produces redundant broadcast messages. This redundancy can become overwhelming in dense wireless networks, leading to a loss of precious bandwidth and battery power and to a dramatic degradation of performance, a situation called “broadcast storm” [1].

Several broadcast schemes have been developed that avoid broadcast storms. The performance of these schemes is measured in terms of *reachability*, that is the fraction of the total nodes that receive the broadcast message, the *number of rebroadcasts*, that is the number of nodes that forward the message, and the *latency*, that is the time between the first and last instant that the broadcast message is transmitted. The set of nodes which forward the broadcast message form the so-called *backbone*. Good broadcast schemes ensure reachability close to 1 and simultaneously a small backbone.

Broadcast schemes are commonly divided into two categories [2], [3]: *deterministic schemes* and *probabilistic schemes*. Deterministic schemes typically build a fixed backbone that guarantees reachability 1 under the assumption of an ideal MAC layer. However, they incur a large overhead in terms of time and message complexity for building and maintaining the backbone, especially in the presence of node failure or mobility. Examples include pruning [4]–[6], multipoint-relaying [7], node-forwarding [8], neighbor elimination [9], [10] and clustering [11], [12].

Probabilistic schemes, in contrast, rebuild a backbone from scratch during each broadcast. Nodes make instantaneous local decisions about whether to broadcast a message or not using information derived only from overheard broadcast messages. Consequently these schemes incur a smaller overhead and demonstrate superior adaptivity in changing environments when compared to deterministic schemes [13]. However, they typically must sacrifice in terms of reachability as a trade-off against overhead.

The probabilistic schemes that have been proposed so far are *probability-based*, *counter-based*, *distance-based* and *location-based* schemes [1], [14], [15]. In probability-based schemes every node that receives the message rebroadcast it with probability P_{thr}. In the other schemes, when a node hears a broadcast message for the first time, it starts a timer of random duration. At the instant this timer expires the node rebroadcasts the message if and only if a “rebroadcast condition” is satisfied. This rebroadcast condition is defined by the different schemes as follows: the node received strictly less than η copies of the message (counter-based), the node did not receive the message from another node at a geographic distance less than d_{thr} (distance-based) the geographical area within the node's range that is yet to be covered by the broadcast exceeds a_{thr} (location-based). There have been several efforts to develop improved probabilistic schemes that choose appropriate thresholds and combine the above mentioned schemes [16]–[18]. The literature hitherto, however, contains very little theoretical analysis of probabilistic schemes.

The first contribution of this paper consists of a novel coloring paradigm for broadcast. Using this paradigm we construct and study an *efficient* probabilistic scheme called *color-based broadcast*. Notably, the coloring paradigm imprints a structure on the backbones which facilitates a rigorous approach of issues regarding the optimality of the afore mentioned schemes. Having such a powerful and general *analytical framework* at the researcher's disposal marks an important step forward in this field where analytical work has been hindered to some extent by the complexity of the space of probabilistic backbones.

In color-based broadcast schemes every broadcast message has a color-field. The rebroadcast condition to be satisfied at the expiration of the timer is similar to the counter-based scheme: the number of *colors* of broadcast messages overheard must be less than *threshold* η. If satisfied, the message is rebroadcast with a new color assigned to its color-field.

We prove that for any given set of backbone nodes B generated by a counter-based scheme there exists a set of backbone nodes B' that can result from the color-based scheme with the same threshold η such that $B \subset B'$. Color-based schemes, however, typically produce backbones with a connectivity graph richer than that of counter-generated backbones. This difference is particularly apparent in the case $\eta = 2$ where the counter-scheme *always* creates a tree-shaped backbone, while the color-scheme almost always creates a mesh-shaped backbone; notably, meshed backbones are more robust to node failure, allow for multipath routing, and reduce latency, all valuable assets in ad hoc routing.

The coloring paradigm brings about a framework which enables an *analytic* approach to studying the number of rebroadcasts and the reachability of color- and counter-based schemes. Using that broadcast nodes of equal color are out of each other's range we prove that color- and counter-based schemes always lead to a backbone of size at most a small constant factor of the optimum. In addition, we compute tight asymptotic bounds for the number of rebroadcasts of color- and counter-based schemes in homogeneous dense networks.[1] Note that probability-based schemes cannot always lead to a backbone of size bounded by a constant factor of the optimum. In probability-based schemes, since nodes independently take decisions to broadcast, large inefficient backbones can often develop.

Increasing η obviously increases reachability on average. A pertinent issue we investigate is a value of η that can guarantee full-reachability for an arbitrary network. We also study the implicit trade-off of reachability against overhead when choosing η; while simple worst case scenarios preclude a general answer valid for any network we find that in homogeneous networks the surprisingly low values $\eta = 3$ and $\eta = 2$ perform well.

The second contribution of this paper consists of two techniques which serve as enhancements to improve the performance of color- and counter-based schemes. The first technique, called *boosting*, uses an initially high "boosted" threshold η which declines with distance to the source node of the broadcast. Boosting drastically reduces the chances of a broadcast dying out after traveling only a few hops away from the source, a situation that occurs frequently with the basic counter- and color-based schemes. This technique improves the reachability significantly at a very modest price in terms of overhead. The second technique, called *edge-growing*, uses information that is implicitly contained in the number and colors of rebroadcasts overheard in order to reduce the number of redundant rebroadcasts. Similar improvement techniques can be designed for other probabilistic schemes.

[1]By *homogeneous dense network* we refer to a network in which the distribution of number of nodes per unit area is the same everywhere and its mean approaches infinity in the limit. From simulations we find that the performance remains the same when mean density is larger than 20 nodes per radio range, assuming ideal MAC and physical layers. In other words, a density of 20 nodes per radio range is practically indistinguishable from the ideal limit of a homogeneous dense network.

Through experiments we evaluate and compare the various, above mentioned novel and enhanced broadcast schemes in terms of reachability and number of rebroadcasts. We do not compare our schemes to distance-based and location-based schemes because we wish to concentrate on the algorithmic advantages of the coloring scheme in this paper. Indeed, these latter schemes employ additional and potentially costly hardware such as GPS to provide more precise information about node locations in lieu of an inexpensive algorithm [1].

We start by proposing color-based broadcast scheme and explaining the closely related counter-based scheme in Section II. Next, we analyze the number of broadcast nodes and the reachability of the color-based and counter-based scheme in Section III. In Section IV, color-based and counter-based schemes are compared via simulation in homogeneous networks. In Section V, we propose two techniques boosting and edge-growing for increasing the reachability and reducing the number of redundant broadcast nodes in the color and counter-based schemes and evaluate them through simulations. Finally, we conclude the paper and discuss future work in VI. All proofs are placed in the Appendix.

II. Color-based Broadcasting

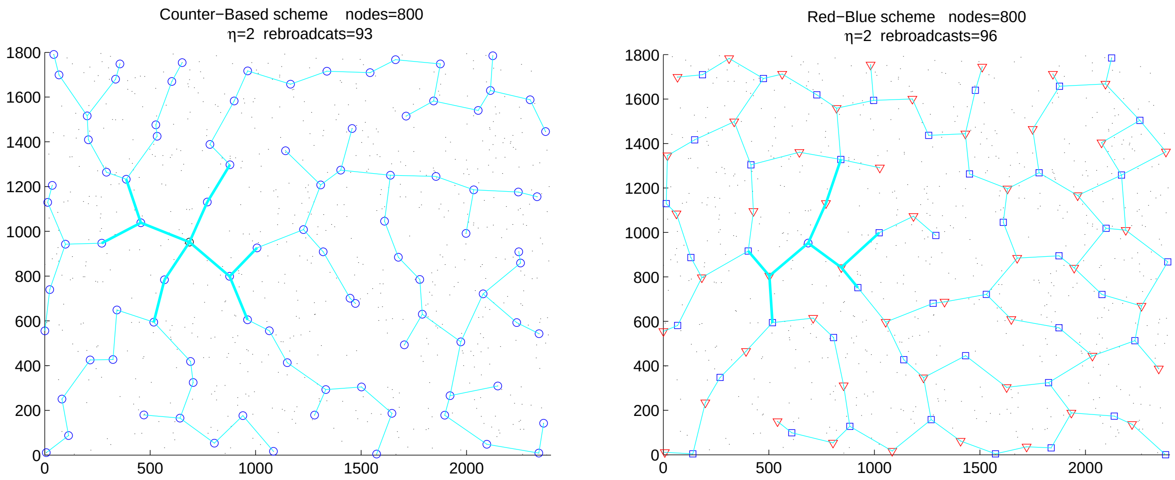
In this section we present novel probabilistic schemes for broadcasting in wireless networks that append colors to broadcast messages. We describe the color-based broadcast algorithm and study the structure of the resulting broadcast backbones which we compare to the backbones generated by counter-based schemes. Our basic *assumptions* for analysis purposes are that the MAC layer is ideal, that the transmission time of the messages is negligible, and that all nodes have the same circular range R.

A. Color-based Broadcast Scheme

The color-based algorithm uses η colors $C_1, C_2, \ldots, C_\eta$. Each broadcast node selects a color which it writes to a *color-field* present in the broadcast message. The algorithm executes in such a way that all nodes which hear the message rebroadcast it unless they have heard all η colors by the time a random timer expires. The set of nodes that broadcast a message, i.e., the backbone randomly varies from one broadcast to the next as a function of the timers.

Color-based broadcast algorithm:

1) The originating node transmits the broadcast message to all neighbors appending the color C_1 to the message.
2) When a node hears a broadcast message for the first time, it waits for a random time interval and records the colors from all broadcast messages overheard during the waiting period. At the end of this period, it compiles the list of colors *not* heard. If the list is not empty, it



(a) (b)

Fig. 1. Random realizations of the backbones for (a) counter-based, and (b) color-based schemes, both with threshold $\eta = 2$. Note the tree-structure of the counter-scheme which contrasts well with the mesh-structure of the color-scheme. While slightly larger than trees, meshes are more robust against failure.

rebroadcasts the message to which it appends the lowest color on this list.

For simplicity in explaining the properties of the scheme, we refer to the color selected by a node for broadcasting a particular message as that node's color.

The backbone of a color-based scheme has several important features. The backbone forms a connected set. However, it is not guaranteed to form a dominating set. Hence color-based broadcast does not guarantee reachability 1 like other probabilistic schemes. By construction nodes with the same color form an independent set. Consequently, colors encode important geometric information.

B. Comparing Color-based and Counter-based Backbones

Recall that the counter-based broadcast algorithm is closely related to the color-based algorithm. In the counter-based scheme, nodes decide whether to broadcast or not based on the number of overheard broadcasts, which must be strictly less than η. In the color-based scheme, node decide whether to rebroadcast based on the number of colors they overhear. Although these two schemes are related, the counter-based scheme cannot be reduced to the color-scheme by a simple change of parameters. A natural question thus arises as to how the backbones generated by the two schemes are related.

The next theorem states that for any given backbone generated by a counter-based scheme there exists a backbone that can result from the color-based scheme with the same threshold η which contains the counter-based backbone as a subset. Vice versa, however, color-based schemes rarely generate a backbone that can be generated by counter-based schemes; more formally, the probability of this happening is very low in settings of practical interest. From experiments we find that color-based schemes typically use more broadcast nodes than their counter-based analogues on average.

Theorem 1: Any backbone generated by a counter-based scheme with threshold η can be colored with η colors such that no two nodes of the same color are within radio range of each other.

As a consequence of Theorem 1, several analytical results of color-based schemes such as Theorems 2 and 3 below extend to the counter-based scheme.

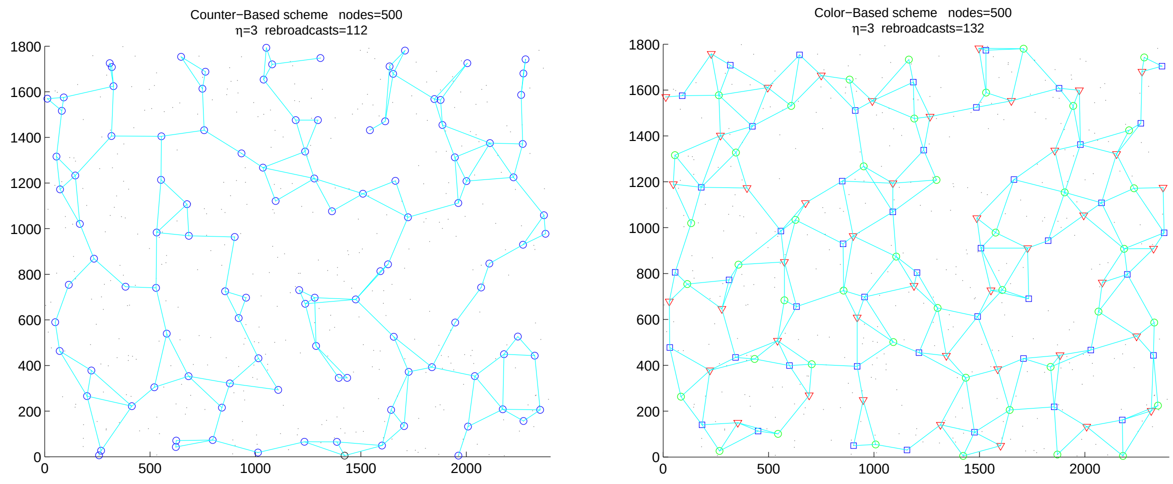
C. Red-Blue Broadcast: the case $\eta = 2$

The relationship between counter- and color-based schemes is particularly simple and explicit for $\eta = 2$ as we elaborate in an instant. Also, we will show in Section IV by simulation that the sweet spot of operation is around threshold values $\eta = 2$ to 3 for homogeneous networks. For these reasons, we give the 2-color scheme the special name *Red-Blue scheme*.

Most notably, a moment's thought shows that a counter-based scheme with threshold $\eta = 2$ *always* generates a backbone with a tree-structure. Indeed, according to the scheme each node in the backbone heard the broadcast exactly once, i.e., from its "parent".

Fig. 1 demonstrates that the Red-Blue scheme generates typically a mesh-like backbone, in contrast to its counter-based cousin. A backbone with mesh-structure is more robust to node failure and leads to lower latency for data transmission; in addition, it can indicate several disjoint paths between the source of broadcast and any fixed node which has immediate implications for ad hoc routing whenever route discovery relies on broadcast.

Reformulating the above more sharply, in the space of all broadcast backbones the counter-based ones are exactly the color-based backbones which happen to be to a tree. Thus, in the case $\eta = 2$ the counter-based scheme can be considered as a color-based scheme with the additional bias to create a tree-shaped backbone. Being trees, the counter-based backbones require slightly fewer broadcasts than the mesh backbones of



(a) (b)

Fig. 2. Random realizations of the backbones for (a) counter-based, and (b) color-based schemes, both with threshold $\eta = 3$.

color-based schemes. However, they are not as robust against failure and mobility as the Red-Blue backbones. This is a consequence of the fact that a tree always gets disconnected by a single node or link failure unlike a mesh.

When increasing the threshold value from $\eta = 2$ to $\eta = 3$, the relation between color- and counter-schemes is no longer so simple. The tendencies of the case $\eta = 2$ prevail, however, also for $\eta = 3$ as is evident from Fig. 2, the color-scheme builds a backbone with a stronger mesh structure than the counter-based scheme. Also, as we should expect, both schemes build backbones with a stronger mesh than with $\eta = 2$ (compare Fig. 1(b) to Fig. 2(b)).

D. Red-Blue Broadcast in Homogeneous Dense Networks

In homogeneous dense networks, the Red-Blue broadcast scheme achieves reachability 1 using a number of broadcast nodes close to the optimum with very high probability. In addition, it builds a mesh structure by a procedure that requires very little overhead. Such a structure can be used later as a backbone for broadcasting, routing or scheduling in the wireless network.

We summarize the interesting properties of the Red-Blue backbones in Proposition 1.

Proposition 1: The subgraph of the network consisting of the Red-Blue backbone has the following properties:

1) The subgraph is connected and bipartite.
2) The subgraph is planar and every face[2] has an even number of edges.
3) The node degree is at most five.
4) The graph has mesh structure with well-spread nodes in a homogeneous dense networks.

We can improve the mesh structure of the color-based backbone via simple modifications of the scheme which we describe next.

[2]A *planar graph* is a graph that can be embedded in the plane so that no edges intersect. A *face* is a region bounded by edges in a planar graph.

Priority-Mesh Red-Blue scheme: We can improve the Red-Blue scheme to build a better mesh structure with a stronger connectivity as follows:

1) By giving priority to nodes that are far from nodes that have already forwarded the broadcast. A measure of distance can be obtained from the signal strength or by adding coordinates to the broadcast. There is a broad literature on obtaining coordinates.
2) By giving higher priority to nodes which hear more broadcasts.

A simple way to implement priority without coordination consists in selecting the average length of the waiting time proportionally to the inverse of distance, respectively to adjust it according to the number of broadcasts heard.

Fig. 3 shows a backbone generated by an enhanced-mesh Red-Blue scheme using only the *second* aforementioned rule in a homogeneous dense network. Observe that the enhanced-mesh Red-Blue scheme builds a mesh with well spread nodes without using any location information of nodes or communicating with other nodes.

III. Theoretical Performance Analysis

In this section we analyze the number of broadcast nodes and reachability of color-based broadcast schemes. As a consequence of Theorem 1, our analysis results can be applied to counter-based schemes as well.

We recall some definitions that will prove useful to describe the properties of the backbones used by different broadcast schemes. An *independent set* is a set of nodes in which no two members are within range of each other. A *connected set* is a set of nodes that cannot be split into two subsets that are out of range of each other. A *dominating set* is a set of nodes such that all nodes in the network are within range of at least one of its members. A *minimum connected dominating set* (MCDS) is a connected dominating set (CDS) with the smallest possible size for a given network. The size of an MCDS equals the

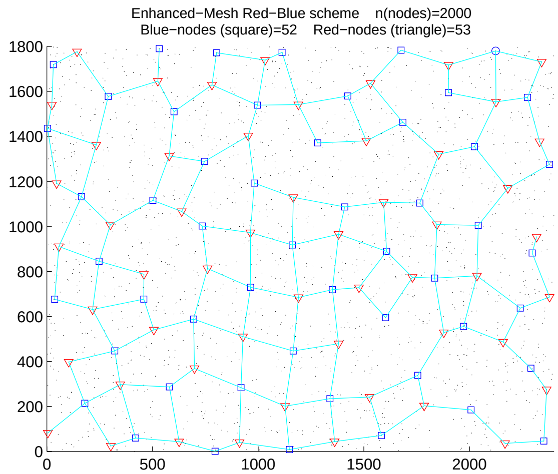


Fig. 3. The enhanced-mesh Red-Blue scheme generates better planar mesh in homogeneous dense networks than the Red-Blue scheme.

minimum number of nodes required for broadcasting to have full reachability. Finding the MCDS of a general wireless ad hoc network is an NP-complete problem [19], [20].

A. Optimality of number of rebroadcasts

The number of nodes used for broadcast in a color-based scheme is bounded by a small constant factor times the MCDS size (see Theorem 2). This property is shared by a number of deterministic broadcast schemes [12], [21], [22]. By nature, these deterministic schemes guarantee reachability 1, unlike the color-based scheme. On the other hand, the deterministic schemes suffer from high overhead in building and maintaining such a CDS in the event of node failure, channel loss and mobility, while the color-based schemes incur almost no overhead to (re)build a CDS for broadcast and to adapt to changing network conditions.

Theorem 2: The number of broadcast nodes in a color-based scheme is at most $\eta(4\#\text{MCDS}+1)$*, where* $\#\text{MCDS}$ *is the size of the MCDS.*

Probabilistic schemes display a very high reachability in homogeneous dense networks. Conditioning on reachability 1, we can bound the number of broadcast nodes in terms of the area and radio range of the nodes. Note that the full-reachability assumption of color-based schemes in dense networks implies that any set of nodes with the same color builds an *independent dominating set* (IDS). Using tight lower and upper bounds for the size of an IDS in a homogeneous dense network, we compute bounds for the number of rebroadcasts of color-based scheme in Theorem 3.

Theorem 3: The number of broadcast nodes in a color-based scheme has the following bounds in a homogeneous dense network.

$$\eta\frac{A}{\pi R^2} < \#\text{rebroadcasts} < 3.6\eta\frac{A}{\pi R^2} \qquad (1)$$

where A *is the area of the network and* R *is the radio range of each node (*$A >> R^2$*).*

B. Reachability

The average reachability of color-based schemes depends strongly on the topology of the network. In a homogeneous dense network a threshold η as low as 2 provides reachability close to 1. However, when the network is sparse η must be set higher to yield similar performance. Experiments show that a threshold of $\eta = 3$ gives very high average reachability even in a sparse homogeneous network.

While increasing η obviously increases reachability on average, the question arises as to whether or not there is a threshold value $\hat{\eta}$ that can provide full-reachability for any arbitrary connected network. Note that our goal is not to find a threshold value that *guarantees* full-reachability for *every* rebroadcast in every connected network. It can be easily shown that such a threshold does not exist. Instead, our goal is to search for a threshold value $\hat{\eta}$ such that for any given connected network, there exists *at least one* random realization of a color-based broadcast backbone which forms a CDS and gives full reachability. Theorem 4 states that 13 is one possible value of $\hat{\eta}$.

Theorem 4: Consider any node n_0 *that originates a color-based broadcast in an arbitrary connected wireless network in which all nodes have equal circular radio range. If* $\eta = 13$*, then among all possible color-scheme backbones there exists at least one which has full-reachability.*

We next illustrate that $\hat{\eta}$ cannot be less than 9 with the help of a pathological example depicted in Fig. 4. The radii of small and large circles are $0.5R$ and $1.5R$ (R is radio range of each node) respectively. By a simple geometric computation we can show that the nodes on the large circle are not connected to each other. As a result, every possible CDS of this network must include all the nodes on the small circle. Since all nodes on the small circle are within hearing range of each other, we need at least 9 different colors to build a CDS of this network using a color-based scheme.

IV. Simulation-Based Performance Analysis

This section studies the performance of counter-based and color-based broadcast schemes in terms of reachability and number of rebroadcasts by simulation. In order to isolate the effects of various design choices of the broadcast algorithms on performance we do not simulate other protocol layers such as the MAC and physical layers.

A. Simulation setup

We simulate a rectangular 2400m by 1800m area populated with nodes that are uniformly distributed in the region, each with circular radio range of radius 250m. This corresponds to networks consisting of a few hundred nodes and roughly 20 radio hops across.

All results we present for reachability and number of rebroadcasts are *averages* computed over several random realizations. For a particular number of nodes in the network, we synthesize 40 topologies and for each topology we randomly choose 50 originating broadcast nodes, a total of 2000 realizations.

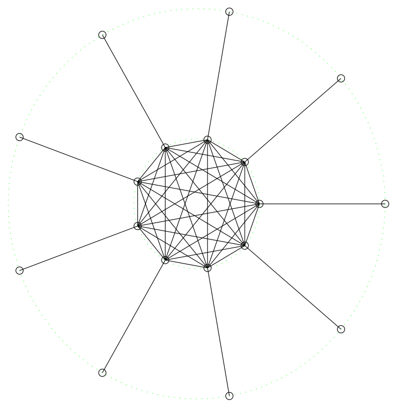


Fig. 4. For coloring the nodes of any CDS of this network, we need a minimum of 9 colors (the small and large circles have radii of $0.5R$ and $1.5R$ respectively).

By changing the total number of nodes in the area we vary the node density. Notably we study not only dense networks which are almost surely fully connected and which are the typical object of study in the literature on probabilistic schemes, but also sparse networks. However, for the experiments to remain meaningful we do not go below densities for which the largest connected component is expected to cover less than half of the given area. Our analysis of sparse networks is robust in the sense that we find a very close agreement between results obtained when we deal with the occurrence of disconnected networks in two ways: (1) considering only those realizations of networks which are fully connected and (2) considering *all* realizations of networks but only the nodes contained in the largest connected component. In this section we present the results which have been obtained by the first method.

B. *Performance comparison of color- and counter-based broadcast*

We start with a comparison of counter- and color-based broadcast schemes. From Fig. 5(a) we conclude that the sweet spot of operation (in terms of reachability beyond 95% at low overhead) lies at threshold 2 for dense networks (from average degree 12 onwards) and at 3 for more sparse networks (from average degree 7 to 12) for the simulated network topology.

As expected the color scheme employs a few more nodes than the counter scheme while providing a more robust mesh structured backbone (see Fig. 5(b)). It does not, however, improve the *average reachability*. The next section provides some first steps towards designing advanced algorithms which cost little more than the $\eta = 2$ algorithms without sacrificing much of the performance of $\eta = 3$ algorithms. By comparing these different advanced schemes, the advantages of color-based schemes over counter-based schemes becomes more clear.

Note that as expected, increasing η for the counter-based or color-based schemes improves reachability at the expense of a larger number of rebroadcasts. While the color-based and the counter-based schemes with threshold $\eta = 2$ perform almost identically, the counter-based scheme with $\eta = 3$ shows greater efficiency in terms of rebroadcasts than the corresponding color-based scheme with the same level of reachability.

V. Improving the probabilistic schemes: Boosting & Edge-growing

In this section we propose two techniques which increase the average reachability and decrease the number of broadcast nodes of probabilistic broadcast schemes. We develop them in detail for color-based and counter-based schemes and provide preliminary outlines of their application to other probabilistic schemes. We study their impact on counter- and color-based schemes through simulation.

A. *Boosting: Increasing Reachability*

A careful analysis reveals that the poor average performance of threshold 2 in sparse networks is due the fact that very often the broadcast covers only a few nodes. To discover why this occurs, we add a hop-count to the broadcast message which is initialized to zero by the node originating the broadcast. When a node rebroadcasts a message it sets the hop-count equal to the lowest hop-count it heard plus 1. Fig. 6(b) reveals that in a high percentage of cases the maximum hop-count is lower than 2, i.e., the broadcast literally “dies out” after two hops.[3] In these cases we see very low reachability (see Fig. 6(a)).

To address this shortcoming we *boost* the count- and color-broadcast as follows: if the hop-count is smaller than N_{boost} use threshold $\eta = 3$, otherwise use $\eta = 2$. Fig. 6(c) and 5(a) demonstrate the considerable improvement achieved by this simple enhancement when $N_{\text{boost}} = 2$. In very few cases the broadcast terminates after two hops thus leading to a significantly higher reachability.

Boosting techniques can be developed for other probabilistic schemes such as probability-, distance-, and location-based broadcast. Analogous to boosting for color-based broadcast, we use different values of parameters for nodes within $N_{\text{boost}} = 2$ hops of the node originating the broadcast and those further away. A detailed study of these techniques is beyond the scope of this paper.

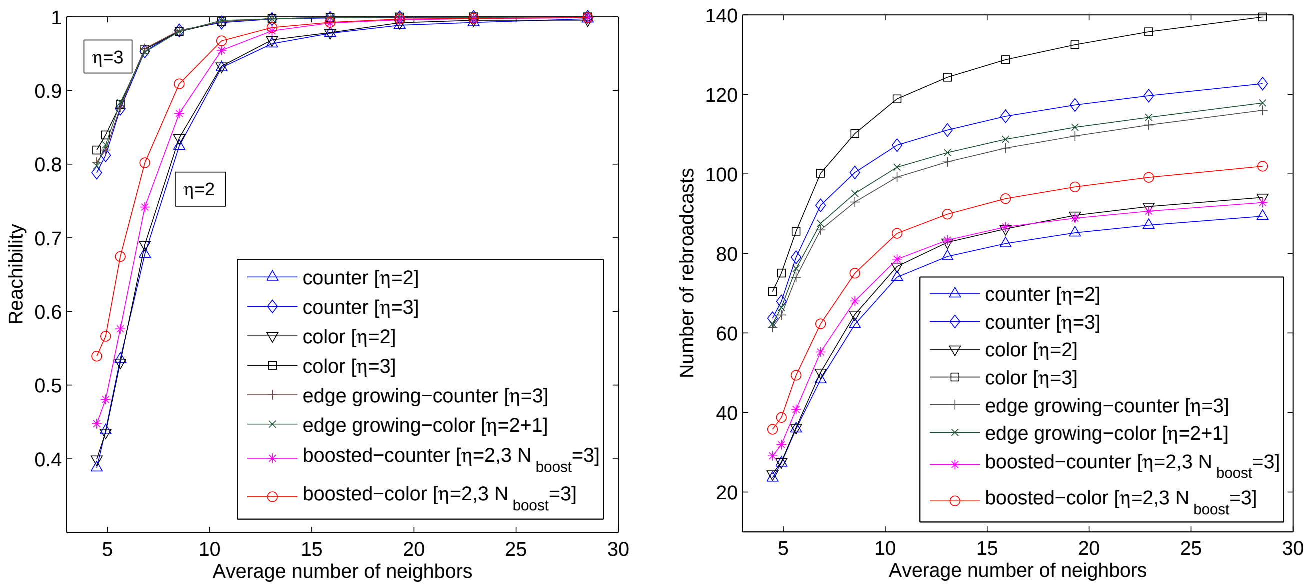
B. *Edge-growing: decreasing the number of rebroadcasts*

Here, we develop schemes that exploit the spatial information implicitly present in overheard broadcast messages more thoroughly to decrease the number of rebroadcasts.

Edge-growing color-based scheme

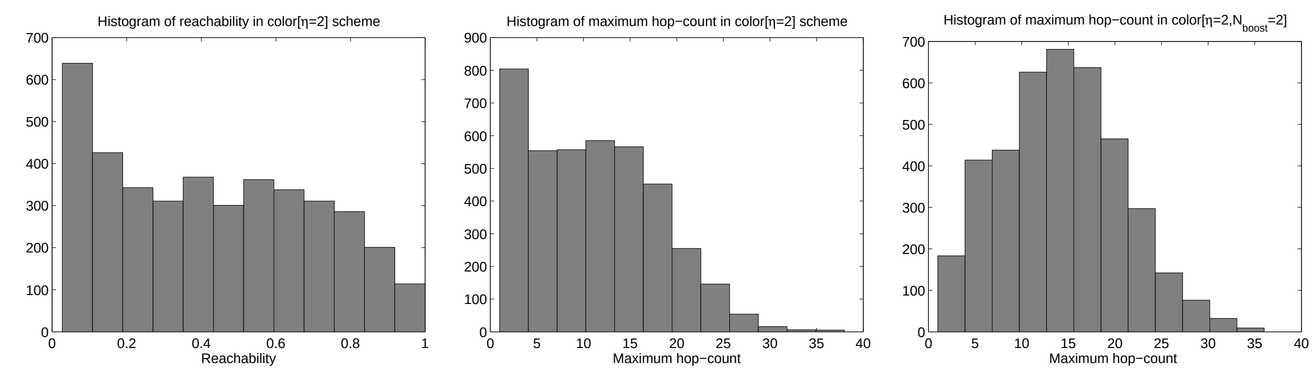
Edge-growing color-based schemes use two sets of colors for broadcasting: η *internal colors* and η' *boundary colors*. The function of internal colors is identical to that of colors used by algorithms described in previous sections. The function of boundary colors is to give nodes on the boundary of the

[3] In the setting of Fig. 6, as a rule of thumb, one hop covers roughly 4% and two hops roughly 10% of the nodes.



(a) (b)

Fig. 5. Comparing different probabilistic broadcast schemes in terms of (a) reachability and (b) number of rebroadcasts.



(a) (b) (c)

Fig. 6. Histograms for a 2-color scheme: (a) reachability, (b) maximum hop-count, (c) maximum hop-count with $N_{\rm boost} = 2$.

area covered by the broadcast at any time instant priority to rebroadcast over other nodes that have heard the broadcast.

Nodes rebroadcast a message using internal colors just as described in Section II. Nodes that do not participate in the broadcast according to the internal colors may still participate using boundary colors according to the following algorithm.

Algorithm

3) (addition to step 2 of the color-based broadcast algorithm) If at the expiration of the random timer of the color-based broadcast algorithm all internal colors have been heard, then the node starts another random timer, called a boundary timer.
4) When this "boundary timer" expires, if the node heard at least β ($2 \leq \beta \leq 5$) nodes possessing the same color or if it heard all of the boundary colors $\{B_1, B_2, ..., B_{\eta'}\}$ then it does *not* rebroadcast the message. Otherwise it rebroadcasts the message to which it appends the lowest color in the list of unheard boundary colors.

The edge-growing color-based broadcast uses typically a smaller number of rebroadcasts than a color-based broadcast scheme using $\eta + \eta'$ colors while providing almost the same reachability. In a color-based scheme that uses $\eta + \eta'$ colors, a node must necessarily rebroadcast when its timer expires if it has not heard all $\eta + \eta'$ colors. In contrast, in the edge-growing color-based scheme a node need not necessarily rebroadcast if it has not heard all boundary colors by the time its timer expires. This results typically in a smaller number of rebroadcasts for the edge-growing color-based broadcast scheme.

Increasing the parameter β leads to an improvement in reachability. Recall that nodes announcing the same color are independent. Thus, as the number of neighbors that a node hears announcing the same color increases, the fraction of uncovered area within its radio range decreases (see Fig. 7). In

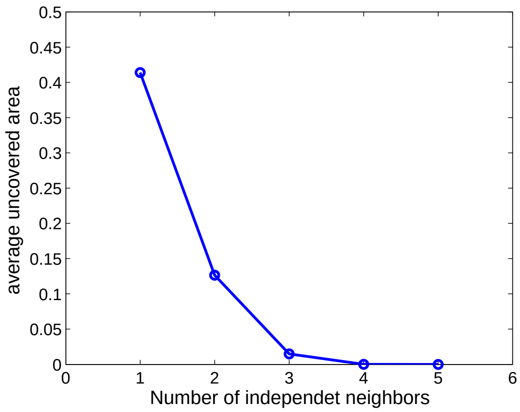


Fig. 7. The average uncovered area within radio range of a node decreases rapidly as the number of *independent* neighbors that rebroadcast increases.

fact, as little as 2% of the radio area of a node is uncovered on average when it has overheard the same color from just 3 neighbors. Through simulations in Section V-C we observe that $\beta = 2$ ensures a high reachability.

Edge-growing counter-based scheme

We propose a novel edge-growing counter-based algorithm of similar flavor as the edge-growing color-based algorithm. This scheme retains the same reachability as the edge-growing color-based algorithm while using fewer rebroadcasts.

The edge-growing counter-based algorithm is identical to the counter-based algorithm described in Section I except that every time a node hears a broadcast message from an additional node it stops its timer and starts a new one with longer mean duration. For example, the mean duration of the random timer of a node that has heard k ($k < \eta$) copies of the message can be set to $T_0 k$ where T_0 is some constant. As a result, nodes that hear fewer rebroadcasts, which is typical of nodes at the outer edge of areas covered by a broadcast, have a higher chance of rebroadcasting the message than those in the interior of the same area. Nodes in the interior of the area covered by the broadcast will hear the message from several nodes and hence not generate redundant rebroadcasts.

Edge-growing techniques can be developed for distance- and location-based broadcast. Similar to edge-growing for counter and color-based broadcast, we give priority to nodes at the boundary of the broadcast region to rebroadcast. Nodes infer that they are on the boundary from the distance or location information of nodes that have already rebroadcast the message.

C. Comparison of Enhanced Color- and Counter-based Schemes

Here we simulate the boosted and edge-growing methods for color-based and counter-based schemes to evaluate how much these methods enhance the performance of the schemes.

We compare the boosted color-based scheme with $N_{\text{boost}} = 3$ to the counter-based scheme with $N_{\text{boost}} = 3$. From Fig. 5 observe that the boosted color-based scheme has a better reachability than the boosted counter-based scheme especially when the network is sparse.

We consider the edge-growing color scheme with 2 internal and 1 boundary color and the edge-growing counter-based scheme with $\eta = 3$. Fig. 5 demonstrates the superiority of the edge-growing schemes: for all node densities considered, the edge-growing $2 + 1$ color-based and the edge-growing 3 counter-based schemes achieve a reachability almost equal to that of the basic counter-based scheme with threshold $\eta = 3$ while using fewer broadcasts.

VI. Summary and Conclusion

A large body of literature discusses and compares existing broadcast schemes extensively. Most of the existing theoretical work has analyzed deterministic algorithms. For probabilistic schemes, little exists as of today beyond experimental studies.

In this paper we presented a novel coloring paradigm for broadcast that provides new color-based broadcast schemes as well as a theoretical framework for analyzing different probabilistic schemes. We showed that assigning colors to broadcast messages provides a simple and effective means to efficiently broadcast messages. Each color defines an independent subset of the backbone which allows rigorous analysis and encodes geometrical information.

The closest existing scheme to our color-based scheme is the counter-based scheme. We proved that any backbone generated by a counter-based scheme is contained in a backbone that can result from the color-based scheme with the same threshold η. We argued that color-based schemes, however, typically produce backbones with a richer connectivity graph than counter-based schemes. We also proved that color and counter-based schemes guarantee a number of rebroadcasts less than a small constant times the optimum.

Experiments with homogeneous networks showed that colors and counters lead to backbones roughly equal in reachability. The sweet spot of operation appears to be between thresholds $\eta = 2$ and $\eta = 3$ in moderate to dense networks. Clearly, higher thresholds might be required for very sparse networks. It should be noted, though, that in sparse networks high thresholds result in backbones similar to those of simple flooding, and little gain can be expected since actual broadcast storms do not occur in such scenario.

We also proposed two techniques that enhance the performance of color- and counter-based broadcast. The first technique, called boosting, addresses the problem that probabilistic broadcasts may die out after only a few hops, especially in sparse networks. We find that boosting is considerably more effective when employed in the coloring approaches than in the counting ones due to spatial diversity enforced on the broadcast nodes of same color. Via the second technique, called edge-growing, we were able to reduce the size of the backbone while maintaining high reachability by leveraging the geometric information intrinsically present in the color and number of broadcasts overheard.

Future work includes evaluating the reduction in overhead achieved by routing protocols when they employ color-based broadcast schemes. The impact of the MAC layer and cross-layer design on probabilistic schemes is also an important issue awaiting further research.

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APPENDIX

Proof of Theorem 1: We assign colors to the broadcast nodes of a counter-based scheme while ensuring that nodes within earshot of each other do not have the same color as follows. Call the i'th node that broadcast the message u_i. Assign color C_1 to the originating node u_1. Assume that u_i, $i \leq n-1$ have been colored, and that we want to color u_n. Because u_n rebroadcasted the message using the counter-based broadcast scheme, it must have heard less than η nodes among u_i, $i \leq n-1$. Therefore, there exists at least one color among the $C_1, \ldots, C_\eta$ colors which u_n has not heard. We assign the lowest of these colors to u_n. Proceeding in this manner we assign colors to all broadcast nodes. □

Proof of Proposition 1: We prove the different claims regarding the subgraph of the red-blue broadcast nodes one-by-one below.

1) There is a path in the subgraph from the origin of broadcast to any node in the subgraph. This proves that it is connected. Also, the broadcast nodes are divided into two groups, red and blue, such that no two nodes of the same color can hear each other. The graph is thus bipartite.
2) Connect the nodes with simple straight lines. Consider two distinct edges (r_1, b_1) and (r_2, b_2) where r_1, r_2 are red nodes and b_1, b_2 are blue nodes. Because nodes with the same color cannot hear each other they must be separated at least by distance R. Thus
$$d(b_1, r_1) + d(b_2, r_2) < R + R < d(b_1, b_2) + d(r_1, r_2) \tag{2}$$
where $d(.,.)$ is the Euclidean distance between the nodes and R is the radio range of each node. The triangle inequality and (2) then prove that the edges (r_1, b_1) and (r_2, b_2) do not cross each other. The graph is thus planar. Because the graph is planar and bipartite, every region must have an even number of nodes.
3) The neighboring nodes of every node have the same color. They hence must form an independent set. By simple geometry we can show that the number of neighbors of any node is at most 5.

□

Proof of Theorem 2: Any set of nodes with the same color is an independent set. It has been proved that the size of an independent set is at most $4\#MCDS+1$ [21], [23]. Because the broadcast nodes are divided into η sets of different colors, the number of broadcast nodes in at most $\eta(4\#MCDS+1)$. □

Proof of Theorem 3:
Upper bound: Consider all nodes with the same color C_i; these form an independent set. As a result, the half range circles around these nodes are disjoint. Let N_i denote the number of nodes with colors C_i. In 1940, L. Fejes Tóth has proved that the hexagonal lattice is indeed the densest of all possible plane packings [24]. Therefore, from the hexagonal lattice we have $N_i \pi (R/2)^2 < \pi\sqrt{3}/6A$. Thus $N_i < 3.6A/\pi R^2$ which proves that $\#\mathrm{broadcasts} < 3.6\eta A/\pi R^2$.

Lower bound: Because the number of broadcast nodes is finite and the network is homogeneous and dense, the set of nodes that did not broadcast form a dense covering over the entire area. Also note that any node that did not broadcast the message heard the message at least η times. It follows that the radio range circles around the broadcast nodes must cover each node in the entire area A at least η times. Thus $\pi R^2 \#\mathrm{broadcast} > \eta A$ which proves that $\#\mathrm{broadcast} > \eta A/\pi R^2$.

□

Proof of Theorem 4: We initially construct a CDS that contains the node n_0 and later prove that we can color it using less than 14 colors. The CDS construction involves three steps.

First, we build an independent dominating set (IDS) of nodes in the network such that it includes node n_0. The IDS can be built by any clustering algorithm described in the literature [11], [21], [25]. We denote the set of IDS nodes by I. Because I forms an IDS, all nodes in the network can hear at least one of its elements. In addition, no two nodes in I can hear each other.

Second, we choose some nodes to form paths that connect the IDS nodes and build a CDS. We call these nodes *connectors* and denote the set of connectors by S'. We refer to the set of CDS nodes as $V' = I \cup S'$, the set of edges formed by connecting adjacent nodes in V' as E'. We term the graph (E', V') the *CDS-graph*.

Third, we eliminate all of the redundant connectors by using the following procedure. If removing a connector from V' and all its corresponding edges from E' results in a graph that is still a CDS, we eliminate it. Otherwise we keep the connector. This procedure continues till we cannot remove any more connectors. We term the remaining set of connectors S, and the resulting CDS-graph $G = (E, V)$ where $V = I \cup S$.

This CDS-graph has an important property that we exploit later in the proof. Note that removing any $u \in S$ from the CDS-graph must result in a graph $G_u = (E_u, V_u)$ with at least two disconnected components. If this were not true then the remaining graph would be connected. In addition it would form a dominating set since $I \subset V_u$. In other words the graph would still be a CDS which we know to be false. Obviously each resulting disconnected component must contain at least one adjacent CDS node of u in V.

We now prove that V can be colored using less than 14 colors such that no two vertices in the CDS-graph (E, V) have the same color. We start coloring the CDS from node n_0 with color C_1. Every node is allowed to take a color if at least

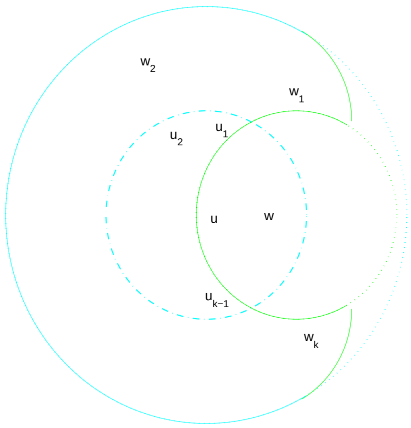

Fig. 8. The location nodes of the built-CDS relative to a connector node u.

one of its neighbors has been colored before. IDS nodes are only allowed to take the color C_1 and connectors any color belonging to the set $\{C_2, C_3, \ldots, C_{13}\}$.

Say that we want to color a node $u \in S$ which has k adjacent connector nodes $u_i \in S$, $i = 1, 2, \ldots, k$ that have already been colored. From the earlier discussion we know that there exists a node $w_i \in V$ adjacent to u_i that does not have a path to u over G_{u_i}. It follows that w_i cannot be in the range of u_j and w_j for all $j \neq i$ because these have paths to u over G_{u_i}. Note that w_j ($j \neq i$) has the path $w_j \rightarrow u_j \rightarrow u$ to u that does not include u_i.

We also know that there exists $w \in V$ adjacent to u that does not have a path over G_u to the n_0. Observe that because u_i has already been colored, it has a path to node n_0 over G_u. Thus w cannot be in the range of any u_i. In addition, since w_i is adjacent to u_i, w cannot be in the range of w_i.

In summary, the set of nodes $\{w, w_1, w_2, \ldots, w_k\}$ are independent and are distributed in the area around u as depicted in Fig. 8. Because any two nodes have distance less than R between them if and only if they are neighbors, for every $j \neq i$

$$d(u_i, w_i) + d(u_j, w_j) < R + R < d(u_i, w_j) + d(u_j, w_i),$$

that is the nodes w_i, u_i, w_j and u_j build a concave quadrilateral.

Through straightforward geometric arguments we can prove that above constraints imply that k is at most equal to 11. Thus the colored connector nodes u_i, $i \leq k$ use at most 11 different colors from $\{C_2, C_3, \ldots, C_{13}\}$ which leaves at least one remaining color that we can assign to u. $\square$

References

[1] Y.-S. C. Sze-Yao Ni, Yu-Chee Tseng and J.-P. Sheu, "The broadcast storm problem in a mobile ad hoc network," in *MobiCom '99*. ACM Press, 1999, pp. 151–162.

[2] B. Williams and T. Camp, "Comparison of broadcasting techniques for mobile ad hoc networks," in *Proceedings of the 3rd ACM international symposium on Mobile ad hoc networking*. ACM Press, 2002, pp. 194–205.

[3] W. Lou and J. Wu, *Localized Broadcasting in Mobile Ad Hoc Networks Using Neighbor Designation*. CRC Press, 2003.

[4] J. Wu and F. Dai, "Broadcasting in ad hoc networks based on self-pruning," in *INFOCOM '3*, vol. 3, 2003, pp. 2240–2250.

[5] F. Dai and J. Wu, "Distributed dominant pruning in ad hoc networks," in *ICC*, 2003.

[6] W. Lou and J. Wu, "On reducing broadcast redundancy in ad hoc wireless networks," in *HICSS '03*. IEEE Computer Society, 2003, p. 305.2.

[7] L. V. A. Qayyum and A. Laouiti, "Multipoint relaying for flooding broadcast messages in mobile wireless networks," in *HICSS '02*, vol. 9. IEEE Computer Society, 2002, p. 298.

[8] P.-J. W. Gruia Calinescu, Ion I. Mandoiu and A. Z. Zelikovsky, "Selecting forwarding neighbors in wireless ad hoc networks," *Mob. Netw. Appl.*, vol. 9, no. 2, pp. 101–111, 2004.

[9] I. Stojmenovic, M. Seddigh, and J. Zunic, "Dominating sets and neighbor elimination-based broadcasting algorithms in wireless networks," *IEEE Trans. Parallel Distrib. Syst.*, vol. 13, no. 1, pp. 14–25, 2002.

[10] M.-I. Stojmenovic, "Comments and corrections to 'dominating sets and neighbor elimination-based broadcasting algorithms in wireless networks'," *IEEE Trans. Parallel Distrib. Syst.*, vol. 15, no. 11, pp. 1054–1055, 2004.

[11] E. Pagani and G. P. Rossi, "Reliable broadcast in mobile multihop packet networks," in *MobiCom '97*. ACM Press, 1997, pp. 34–42.

[12] W. Lou and J. Wu, "A cluster-based backbone infrastructure for broadcasting in manets," in *IPDPS '03*. IEEE Computer Society, 2003, p. 221.1.

[13] P. Rogers and N. Abu-Ghazaleh, "Towards reliable network wide broadcast in mobile ad hoc networks," 2004. [Online]. Available: http://www.citebase.org/cgi-bin/citations?id=oai:arXiv.org:cs/0412020

[14] J. Y. H. Zygmunt Haas and L. Li, "Gossip-based ad hoc routing," Tech. Rep., 2001.

[15] D. C. Yoav Sasson and A. Schiper, "Probabilistic broadcast for flooding in wireless mobile ad hoc networks," in *Proceedings of IEEE Wireless Communications and Networking Conference*, 2003.

[16] S.-Y. N. Y.-C. Tseng and E.-Y. Shih, "Adaptive approaches to relieving broadcast storms in a wireless multihop mobile ad hoc network," in *IEEE Transactions on Computers, vol. 52, no. 5,*, 2003, pp. 545–557.

[17] C.-C. Yang and C.-Y. Chen, "A reachability-guaranteed approach for reducing broadcast storms in mobile ad hoc networks," in *Proceedings of IEEE Semiannual VTC '02*, 2002.

[18] J. Cartigny and D. Simplot, "Border node retransmission based probabilistic broadcast protocols in ad-hoc networks," in *HICSS '03*. IEEE Computer Society, 2003, p. 303.

[19] N. F. Huang and T. H. Huang, "On the complexity of some arborescences finding problems on a multihop radio network," *BIT*, vol. 29, no. 2, pp. 212–216, 1989.

[20] M. V. Marathe, H. Breu, H. B. Hunt III, S. S. Ravi, and D. J. Rosenkrantz, "Simple heuristics for unit disk graphs," *Networks*, vol. 25, pp. 59–68, 1995.

[21] P.-J. W. K. Alzoubi and O. Frieder, "New distributed algorithm for connected dominating set in wireless ad hoc networks," in *HICSS'02-Volume 9*. IEEE Computer Society, 2002, p. 297.

[22] P.-J. W. Khaled M. Alzoubi and O. Frieder, "Message-optimal connected dominating sets in mobile ad hoc networks," in *MobiHoc '02*. ACM Press, 2002, pp. 157–164.

[23] K. M. A. Peng-Jun Wan and O. Frieder, "Distributed construction of connected dominating set in wireless ad hoc networks," *Mobile Network Applications*, vol. 9, no. 2, pp. 141–149, 2004.

[24] D. Wells, *The Penguin Dictionary of Curious and Interesting Numbers*. Middlesex, England: Penguin Books, 1986.

[25] S. Basagni, "Distributed clustering for ad hoc networks," in *ISPAN '99*. IEEE Computer Society, 1999, p. 310.