Bounds on the Benefit of Network Coding for Wireless Multicast and Unicast

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Abstract—In this paper, we explore fundamental limitations of the benefit of network coding in multihop wireless networks. We study two well-accepted scenarios in the field: single multicast session and multiple unicast sessions. We assume arbitrary but fixed topology and traffic patterns for the wireless network. We prove that the gain of network coding in terms of throughput and energy saving of a single multicast session is at most a constant factor. Also, we present a lower bound on the average number of transmissions of multiple unicast sessions under any arbitrary network coding. We identify scenarios under which network coding provides no gain at all, in the sense that there exists a simple flow scheme that achieves the same performance. Moreover, we prove that the gain of network coding in terms of the maximum transport capacity is bounded by a constant factor of at most π in any arbitrary wireless network under all traditional Gaussian channel models. As a corollary, we find that the gain of network coding on the throughput of large homogeneous wireless networks is asymptotically bounded by a constant. Furthermore, we establish theorems which relate a network coding scheme to a simple routing scheme for multiple unicast sessions. The theorems can be used as criteria for evaluating the potential gain of network coding in a given wired or wireless network. Based on these criteria, we find more scenarios where network coding has no gain on throughput or energy saving.

Index Terms—Network coding gain, multicast throughput, energy consumption, transport capacity

1 Introduction

In recent years, network coding has become an important research topic in network information theory. It has been shown that network coding can help to improve the throughput and energy consumption of communication networks. Theoretical studies of network coding provide guidelines for designing or improving efficient and high-performance wired or wireless networks. In this paper, we present theory leading to fundamental bounds on the gain of network coding in arbitrary wireless networks in the sense that there exists a simple flow scheme that achieves the claimed performance without the use of network coding. In particular, our work reveals that the benefit of network coding is limited by constants depending only on the physical dimension of the network, i.e., 1, 2, or 3.

Today, studies on network coding have grown to large numbers, mostly focussing on benefits in terms of throughput or energy saving. In this paper, we study the fundamental limitations of network coding in the same terms and identify scenarios where the network coding gain on the performance is noticeably small. To the best of our knowledge, this is the first paper which studies the bounds on the gain of network coding in terms of energy saving in wireless networks and which presents a framework for relating network coding schemes to simple routing schemes.

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Manuscript received 24 Feb. 2012; revised 16 Aug. 2012; accepted 12 Oct. 2012; published online 9 Nov. 2012.
For information on obtaining reprints of this article, please send e-mail to: tmc@computer.org, and reference IEEECS Log Number TMC-2012-02-0091. Digital Object Identifier no. 10.1109/TMC.2012.234.

The techniques of our arguments include using cutsets, a basic tool in information theory, as well as geometric graph models for the wireless network and representations of unicast transmissions as vectors. In Section 5.3, we convert the problem of network coding of multiple unicast flows to a directed graph and apply the famous max-flow min-cut theorem to obtain novel bounds on the gain of network coding for both throughput and energy saving. The main challenge consisted in choosing an appropriate network model and in ideally combining different mathematical tools from information theory, graph theory, geometry, and algebra.

Our results apply to any network topology, to most relevant channel models and to arbitrary traffic patterns. To this end, our study assumes fixed but arbitrary settings of these parameters and concerns itself with the possible gain to be achieved using network coding as compared to ordinary routing schemes.

As the first result, we prove that the *energy gain* of network coding for a single multicast session is bounded by a constant factor of 7 in an arbitrary wireless network. Also, we bound the *throughput gain* of network coding for a single multicast session in an arbitrary wireless network. We show that network coding can improve the throughput in wireless networks by at most a constant factor which is determined by the parameters of the underlying wireless channel model. This lies in stark contrast to wired networks where the network coding gain in terms of energy consumption and throughput can be an unboundedly large factor [1].

Concerning the *energy gain* for multiple unicast sessions, we provide a novel lower bound on the average number of transmissions for transporting unicast packets for multiple unicast sessions in an arbitrary wireless network. Assuming that each transmission carries the same size packet and that energy consumption is proportional to the number of transmissions, we find, thus, an upper bound on the energy gain of network coding. In addition, we identify important scenarios where network coding benefits in terms of energy consumption constitute a relatively small amount, in some cases *none at all*, for example, for certain traffic patterns in wireless sensor and mesh networks.

For the *throughput* of multiple unicast sessions, we establish that network coding can increase the maximum *transport capacity* by at most a constant factor π in an arbitrary wireless network. From this, we conclude that the traditional bounds on the transport capacity [2], [3], [4], [5] would increase by at most a factor of π if network coding is employed. We also prove that the gain of network coding on the throughput of large random wireless networks is asymptotically bounded by a constant factor. This result is more general than the result of previous work [6], [7] which has been proved for a particular topology and wireless channel model.

As a further outcome of our work we establish theorems that derive relations between network coding schemes on multiple unicast sessions with simple routing schemes (without coding). These theorems can provide criteria for evaluating the potential gain of network coding in a given wired or wireless network. Then, using these theorems, we identify several scenarios where a simple routing scheme is the most efficient scheme in terms of throughput or energy saving among all network coding schemes in the network. Therefore, network coding has no potential gain on the throughput and energy saving in those scenarios.

In summary, we provide several important novel insights, especially on wireless sensor and mesh networks, and generalize to arbitrary networks several results that had been known only for special cases. Doing so, we greatly extend results on the gain of network coding which were established only under very special conditions (such as [6], [7], see Section 2 for details) and provide several novel results.

The paper is organized as follows: In Section 2, we review some related work. We introduce the network model and notations in Section 3. We bound the network coding gain in terms of energy and throughput for a single multicast in Section 4. Next, in Section 5, we study the benefit of network coding in wireless networks for multiple unicast sessions. Finally, we conclude the paper in Section 6.

2 Related Work

Network coding was first introduced in the seminal paper by Ahlswede et al. [8] in which it was proven that the maximum flow capacity of a single multicast session can be achieved using network coding in an arbitrary wired network with directional links. Later, Li et al. [9] and Koetter and Medard [10] show constructively that the linear network codes can achieve the capacity of a single multicast session as well. Since then, a large body of work has explored the construction of efficient network coding algorithms, for example, [1], [11], [12], [13], [14], [15], [16].

For the networks with undirected links, Li et al. [17], [18] show that network coding improves the throughput by at most a constant factor 2 for a single multicast. The constant factor turns out to be equal to one (no benefit) in the case of a single unicast or a broadcast. In recent works, some more general and parameterized models of undirected networks are introduced in [19] and [20]. Some new bounds on the gain of network coding for multicasting in parameterized networks are computed in [19], [20], and [21]. Note that these results do not extend for wireless networks where the channel is considered bidirectional. This is due to the fact that network coding combined with wireless broadcasting can potentially improve the performance in terms of throughput, energy efficiency, and congestion control in wireless networks [22], [23].

The potential of network coding for energy savings in broadcasting in Ad Hoc networks was studied in [24], [25], and [26]. Interestingly, Wu et al. [27] prove that linear network codes always achieve the minimum energy required for transmitting packet in a single multicast session. Here, we complement this valuable insight by showing in that such a gain is in fact bounded by a small constant for a multicast session. Moreover, our work strengthens the results of [28] for the maximum flow achievable in random wired and wireless networks (modeled as geometric random graphs) for a single multicast session by taking wireless interference into account.

The case of multiple unicast sessions in wireless networks was studied in [29] and [30]. The papers studied cases where network coding would provide only marginal benefits. More recently [31], [32], [33] proposed network coding algorithms for improving the throughput and energy consumption of unicast sessions in wireless networks.

Taking a different approach, the papers [6], [7] derive bounds on the potential gain of network coding on the unicast capacity and multicast capacity of large homogeneous wireless Ad Hoc networks with random traffic pattern. These papers prove that that the gain is asymptotically bounded by a constant factor, as the network is scaled to large size. We highlight that in this paper we provide new results which are nonasymptotical and valid for any network. Also, we present several novel bounds on the gain of network coding for energy saving in wireless Ad Hoc networks.

Some papers assume correlation between the generated information from different sources in the network. They combine network coding with distributed source coding (DSC) techniques to optimize the coding gain in the network [34], [35], [36]. However, in this paper, we focus on pure network coding gain. In other words, we assume that the sources generate independent information.

Other work [37] shows that network coding can bring a benefit in large wireless networks if cooperative communication techniques (such as beam forming) are employed [37]. We will not look at such networks.

In summary, the only work with similar assumptions is [6], [7]. However, its results are only asymptotically valid and for uniformly placed nodes with uniform traffic. We greatly extend these results to arbitrary networks (small or large) and arbitrary traffic. As a corollary of our work (see

Corollary 5.5) we confirm the result of [6] and [7] in their very specific settings, however, using entirely different arguments and methodologies. Besides this corollary, all results are novel.

3 Model Assumptions

In this section, we describe the models and notions used in this paper.

3.1 Network Model

A communication network is a collection of directed (or undirected) links connecting communication devices (nodes). The links can be established through actual wired or wireless transmissions. Here, we consider an arbitrary wireless networks. We assume that all channels are broadcast and bidirectional in the wireless network.

We emphasize that we consider an *arbitrary* topology for the wireless network. However, we assume that the topology has the *connectivity* which is needed for establishing the demanded multicast or unicast sessions. We assume that the nodes are distributed in d-dimensional euclidean space $\mathbb{R}^d$ but fixed.

3.2 Wireless Channel Models

We employ traditional Gaussian channel models, namely, the protocol and the physical model [2] for modeling the wireless channel. We denote the set of transmitter-receiver pairs of *simultaneous* transmissions active at a given time instance by $\mathcal{SD} := \{(S_1, D_1), (S_2, D_2), \ldots, (S_m, D_m)\}$. Also, we denote the set of transmitters by $\mathcal{S} := \{S_1, \ldots, S_m\}$. Note that these sets vary over time; if not otherwise indicated, however, we will consider one fixed but arbitrary time instant. For simplicity of notation, the node symbols are used also to represent their locations. For example, $|S_i - D_i|$ is the *euclidean distance* between the nodes S_i and D_i in $\mathbb{R}^d$.

In both, the protocol and the physical model the assigned transmission rate from node $S_i \in \mathcal{S}$ to node D_i is $W_i = W$ for a successful transmission where W is the *channel capacity*; for unsuccessful transmissions $W_i = 0$. Note that we assume a broadcast channel for wireless networks, so a transmission will typically be received by several nodes simultaneously. On one hand, broadcasting data to all neighbors can help to increase the throughput, on the other hand, simultaneous reception from different nodes is not feasible because of interference.

3.2.1 Protocol Model

Under the protocol model a transmission is modeled as successful if $|S_i - D_i| \le r$ and $|S_k - D_i| \ge (1 + \Delta)r$ for all $S_k \in \mathcal{S}\backslash\{S_i\}$, where $\Delta > 0$ is the *interference parameter* and $r > 0$ the *transmission range*.

3.2.2 Physical Model

Under the physical model a transmission is modeled as successful if

$$\mathrm{SINR} = \frac{PG_{ii}}{N_o + \sum_{k \neq i, k \in \mathcal{S}} PG_{ki}} \ge \beta. \tag{1}$$

Here, β is the SINR-threshold, N_o represents the ambient noise, and G_{ki} denotes the signal loss, meaning that PG_{ki} is the receiving power at node D_i from transmitter S_k. We assume a power-law decay for the signal loss of the form $G_{ki} = |S_k - D_i|^{-\alpha}$, where $\alpha > d$ is the signal loss exponent.

We introduce the parameter $r_{\max}$ as the maximum possible distance between a transmitter and receiver which still ensures a successful transmission under the physical model. Equation (1) implies that $r_{\max} = (\frac{P}{\beta N_o})^{1/\alpha}$. Note that $r_{\max}$ is different than the radio range parameter r in the protocol model. Here, a transmitter can achieve the range of $r_{\max}$ only if no other simultaneous transmission occurs in the network. We will use the parameter $r_{\max}$ as transmission range (corresponding to the best case scenario) to compute the minimum energy needed for transporting information. However, we apply (1) for analyzing the maximum throughput (that usually occurs under a set of simultaneous transmissions) in wireless networks.

3.3 Connectivity Graph and Traffic Pattern

Wireless networks are usually modeled by *geometric graphs*. The nodes of network are the vertices $\mathcal{V}$ of geometric graphs. Two nodes are considered adjacent and connected by an edge $e \in \mathcal{E}$, if the distance between them is less than a certain value r_G. We build the *connectivity graph* of a given wireless network by setting $r_G = r$ for the protocol model and $r_G = r_{\max}$ for the physical model in the geometric graph model. We represent the connectivity graph by $\mathcal{G} = (\mathcal{V}, \mathcal{E})$. It is straightforward to show that to establish some sessions in the wireless network there must exist a path in $\mathcal{G}$ between the source and the terminals of each session.

We represent multiple unicast sessions by the set of source-terminal pairs $\mathcal{AB} := \{(A_1, B_1), \ldots, (A_k, B_k)\}$ where $\mathcal{A} := \{A_1, \ldots, A_k\}$ and $\mathcal{B} := \{B_1, \ldots, B_k\}$ are the sets of sources and terminals. We refer to the *hop-count distance* of two nodes A_i from B_j by $\ell(A_i, B_j)$. For the networks with directed links, $\ell(A_i, B_j)$ is the hop-count length of the shortest directed path from A_i to B_j. While the computation of the shortest paths in real-world scenarios is often hampered by incomplete information or other restrictions, using shortest paths is certainly appropriate when deriving theoretical bounds, as they constitute the best of worlds in terms of routing. Also, we define the distance of B_j from a set of nodes $\mathcal{A}$ as usual as $\ell(\mathcal{A}, B_j) = \min_{A_i \in \mathcal{A}}(\ell(A_i, B_j))$. Since topology and channel are not random, there is no randomness in the connectivity graph. We also assume no randomness in the traffic pattern.

3.4 Transport Capacity

The *transport capacity* constitutes a performance parameter of wireless networks, of central importance as it reflects the maximum sum of the rates and the number of transmissions of unicast sessions. Note that the unit of transport capacity is "bit-meter per second" which is different from the unit of throughput capacity (bit per second). Interestingly, by computing the transport capacity of wireless network one can estimate the average rate of the unicast sessions when the average distance of sources and terminals is given. The transport capacity of a set of source-terminal pairs $\mathcal{AB}$, is defined as

$$C_T(\mathcal{AB}) := \max_{\text{multihop paths}} \sum_k |A_k - B_k| R_k, \tag{2}$$

where R_k is the average rate of unicast session between A_k and B_k over a given multihop path. The maximum is taken over all possible multihop routes establishing the required connections between the sources and terminals. A simple upper bound which actually does not depend on the set $\mathcal{AB}$ is found by noting that for the simultaneous routes achieving C_T there must be a time instance where the simultaneous direct transmissions reach at least C_T [2]:

$$C_T(\mathcal{AB}) \leq \max_{\mathcal{SD}} \sum_{(S_i,D_i)\in\mathcal{SD}} |S_i - D_i| W_i, \tag{3}$$

where the maximum is over all possible sets of simultaneous transmissions $\mathcal{SD}$, also W_i is the transmission rate of (S_i, D_i) transmitter-receiver pair. The transmission rate is computed based on the channel model.

3.5 Flow Schemes and Network Coding

In our arguments, we will use the term *flow scheme* to denote a simple noncoding scheme that route as commodity flows (replication, forwarding). In contrast, we denote by a *coding coding* any scheme using all of the operations of a flow scheme and in addition allowing the packets to be coded or recoded at each node where they are received. In a network coding scheme, intermediate nodes can send the results obtained from applying arbitrary functions to all previously received data and their own source data with the only restriction that each destination node is able to decode the data intended for it from all of its received bits and from local data.

3.6 Data Streams and Energy Consumption Model

In this paper, we assume that the generated data from every source is optimally source coded. Moreover, the data of different sources are independent from each other. Agreeably, this manifests a simplifying yet appropriate assumption. Indeed, assuming some dependence among the generated information leads naturally to DSC techniques for maximizing the throughput [34], [35]. However, the focus of this paper lies in pure *network coding gain* and not in source coding gain, be it single nor multiple source coding gain.

We quantize the *consumed energy* for transporting information by assuming that every transmission sends a constant amount of information, i.e., c bits. Consequently, the consumed energy for transporting the information becomes proportional to the *number of transmissions*. Note that we assume that each transmission consumes the same amount of energy, independently of distance.

As a particular consequence of our setting, we assume that the header of the packets is not used as information, so the only information (c bits) that is transmitted is the content of each transmission. Consequently, we do not consider the overhead provided by routing, nor potential ways to exploit routing information. In addition, we assume that the information cannot be communicated between the nodes without transmission, in other words, timing or omission of transmissions does not provide any new information to the receiver nodes. This can give us a good estimation of energy consumption in real wireless networks when the packet size is large compared to the header size, for example, IEEE 802.11. Note that we also do not consider the energy consumption used for carrier sensing, reception, MAC control packets in wireless communication.

The above assumptions imply that the *average number of transmissions* for sending M specific information bits between two parts of the network (without any loss) is on average at least M/c transmissions per input packet.

For example, assume $c = 1$ and two received bits 00, 01, 10, and 11 may be coded as 0, 10, 110, and 111. Thus, the two received bits 00 are sent on as 0 and the number of transmissions is reduced for this instance. However, the average number of bits received at each terminal cannot be reduced since the bits are optimally compressed (at maximum entropy). In this example, we observe that, on average, $((1 + 2 + 3 + 3)/4)/2 = 9/8$ bits are sent per bit.

Short, each terminal must receive at least 1 bit per bit sent, on average, over the entire message. Since we establish only upper bounds on the gain or cost, thus for some of our analysis we consider just one packet that is received at the terminals.

We recall that all our settings (connectivity graph, traffic pattern, etc.) are fixed, but arbitrary. No averages other than the average over the bits of the message are taken.

4 Bounds on the Gain of Network Coding for a Single Multicast Session

In this section, we bound the benefit of network coding in terms of energy savings and throughput for a single multicast session in wireless networks.

4.1 Energy Gain of Network Coding in Wireless Networks

In Theorem 4.3, we show that the number of transmissions can be reduced by at most a constant factor for a single multicast session when a network coding scheme is employed.

Our rationale consists of the following steps. Certainly, a network coding scheme can reduce the number of transmissions of a multicast session. Nevertheless, the mutual location of source and terminals within the network topology together with the geometric properties of the wireless channel models enforce a minimal number of transmissions for transporting the packets under any arbitrary network coding. These minimal transmissions establish certain communication paths which can be exploited by a flow scheme to deliver the same information to the same terminals with just a constant factor more transmissions. As we show, this factor is surprisingly small.

For a rigorous argument, we introduce the following topological parameters. Let $\mathcal{M}$ be the set of all terminals together with the source of the single multicast session. Next, let $\mathcal{C}$ be a set of nodes with minimum size which includes the source and covers all terminals, i.e., every node of $\mathcal{M}$ is either in $\mathcal{C}$ or has at least one neighbor in $\mathcal{C}$.

In addition, iteratively let $\mathcal{H}_i = \{u \in V : \ell(u, \mathcal{H}_{i-1}) \leq 1\}$ for $i \geq 1$, where $\mathcal{H}_0 := \mathcal{M}$. Clearly, by the definition $\mathcal{H}_0 \subseteq \mathcal{H}_1 \subseteq \mathcal{H}_2 \subseteq \dots$ (see Fig. 1). Further let n_i be the number of connectivity components of the graph created by the nodes of $\mathcal{H}_i$ (assuming that the nodes of $\mathcal{V}\backslash\mathcal{H}_i$ are removed from the network graph $\mathcal{G}$). Also, let $k := \min\{j : n_j = 1\}$. Clearly, $n_0 \geq n_1 \geq \dots \geq n_k = 1$.

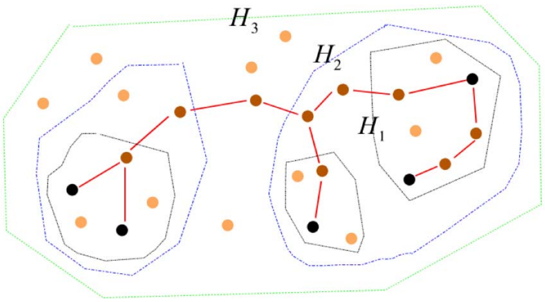

Fig. 1. The nodes of $\mathcal{M}$ have been colored black. The figure shows $\mathcal{U}$ which is built in three steps by considering the components of $\mathcal{H}_1$, $\mathcal{H}_2$, and $\mathcal{H}_3$.

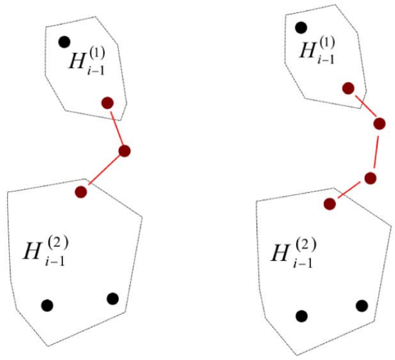

Fig. 2. Two components $\mathcal{H}_{i-1}^{(1)}$ and $\mathcal{H}_{i-1}^{(2)}$ are connected after building $\mathcal{H}_i$ if and only if one of these two scenarios occurs.

Next, we establish two lemmas which will be instrumental for the proof of Theorem 4.3.

Lemma 4.1. *Denote the average number of transmissions for sending a multicast packet under an arbitrary network coding scheme by N. Then,*

$$N \geq \max(|\mathcal{C}|, n_1 + \cdots + n_k), \tag{4}$$

where $|\mathcal{C}|$ is the number of nodes in $\mathcal{C}$.

Proof of Lemma 4.1. We prove the lemma in two parts.

1. *Proving that $N \geq |\mathcal{C}|$.* By assumption all terminals receive the packet that is multicast by the source using altogether N transmissions. The set of nodes that are used for multicasting must contain the source and covers all terminals; also, its average size is N nodes. By the minimality property of $\mathcal{C}$, we can conclude that the number of transmissions N is at least equal to $|\mathcal{C}|$.
2. *Proving that $N \geq n_1 + \cdots + n_k$.* Consider the cutset between $\mathcal{H}_{i-1}$ and $\mathcal{V} \setminus \mathcal{H}_{i-1}$ in the network graph $\mathcal{G}$. We claim that at least n_i transmissions are needed in this cutset for disseminating a multicast packet through all connectivity components of $\mathcal{H}_{i-1}$. Notice that each component of $\mathcal{H}_{i-1}$ includes either the source or some terminals or both, therefore, there must be at least one transmission from/toward each component. Otherwise, the multicast packet cannot be fully decoded by the terminals of some components. The transmission nodes of this cutset belong to $\mathcal{H}_i \setminus \mathcal{H}_{i-1}$, and since $\mathcal{H}_i$ has n_i connectivity components the number of transmissions cannot be less than n_i.

Note that $\mathcal{H}_0 \subset \mathcal{H}_1 \subset \cdots \subset \mathcal{H}_k$. This means that the mentioned cutsets are disjoint for $i = 1, \ldots, k$. Therefore, the total number of transmissions for transporting a multicast packet is at least $n_1 + \cdots + n_k$. □

Lemma 4.2. *There exists a flow scheme that transports a multicast packet using N' transmissions, where*

$$N' \leq 5|\mathcal{C}| + 2(n_1 + \cdots + n_k) - 2k - 6. \tag{5}$$

Proof of Lemma 4.2. The idea of the proof is to build a suboptimal Steiner multicast tree using $\mathcal{H}_i$'s, i.e., we find a set of nodes $\mathcal{U}$, such that the size of $\mathcal{U}$ is equal to N' and that the induced graph by $\mathcal{U}$ is connected, contains the source node, and covers all terminals. Then a flow scheme which uses the nodes $\mathcal{U}$ can disseminate the multicast packet from the source to all terminals.

We construct $\mathcal{U}$ in $k+1$ steps. First, we set $\mathcal{U}_0 = \mathcal{C}$. Notice that $\mathcal{U}_0$ contains the source and covers all terminals, so for building $\mathcal{U}$ it is sufficient to connect the nodes of $\mathcal{U}_0$ together.

For $1 \leq i \leq k$, we iteratively construct $\mathcal{U}_i \subseteq \mathcal{H}_i$ by adding the minimal number of nodes to $\mathcal{U}_{i-1}$ to reduce the number of its connectivity components to n_i. The set $\mathcal{U}_k$ is our target set $\mathcal{U}$.

Now, we bound the number of nodes which is added to $\mathcal{U}_{i-1}$ at step i. Without loss of generality, we may assume that $n_{i-1} > n_i$. This means $n_{i-1} - n_i$ connections must be created among the components of $\mathcal{U}_{i-1}$ after building $\mathcal{U}_i$. We will show that at most $(2i+2)(n_{i-1} - n_i)$ nodes are needed for creating the connections.

As an illustrative example, consider $\mathcal{U}_{i-1}^{(1)}$ and $\mathcal{U}_{i-1}^{(2)}$ as two components of $\mathcal{U}_{i-1}$ which must get connected after building $\mathcal{U}_i \subseteq \mathcal{H}_i$. Notice that two components are connected as we build $\mathcal{U}_i$ from $\mathcal{U}_{i-1}$ only if one of the depicted scenarios in Fig. 2 occurs. It follows from the definition of $\mathcal{H}_i$ that there exists a path of length $2i$ or $2i+1$ between one node in $\mathcal{M} \cap \mathcal{U}_{i-1}^{(1)}$ and another one in $\mathcal{M} \cap \mathcal{U}_{i-1}^{(2)}$.

We add the nodes of this path to $\mathcal{U}_{i-1}$ (the number of nodes of the path is at most $2i+2$). Since $\mathcal{U}_0 = \mathcal{C}$ is a cover set for $\mathcal{M}$, two nodes on $\mathcal{U}_0$ which are in $\mathcal{U}_{i-1}^{(1)}$ and $\mathcal{U}_{i-1}^{(2)}$ are connected by the constructed path. This process is repeated $n_{i-1} - n_i$ times to connect all the corresponding components of $\mathcal{U}_{i-1}$. Since at most $2i+2$ new nodes are added each time, we will use at most $(2i+2)(n_{i-1} - n_i)$ nodes at step i.

So, by continuing this algorithm and increasing i, the set $\mathcal{U}_i$ gets completely connected in the kth step. Based on the construction, the size of $\mathcal{U}$ is bounded by

$$|\mathcal{C}| + (2+2)(|\mathcal{C}| - n_1) + \sum_{i=2}^{k}(2i+2)(n_{i-1} - n_i)$$
$$= 5|\mathcal{C}| + 2(n_1 + n_2 + \cdots + n_{k-1} + n_k) - 2k - 6.$$

□

Theorem 4.3. *The gain of network coding in terms of reducing the number of transmissions of a single multicast session is less than a factor of 7.*

Proof of Theorem 4.3. Lemmas 4.1 give us a lower bound on the number of transmissions (N) under any arbitrary network coding scheme. Also, Lemma 4.2 shows that there exists a flow scheme that uses N' transmissions per multicast packet. It follows that the benefit of network coding in terms of reducing the number of transmissions is bounded by N'/N. From (4) and (5), it is straightforward to show that $N'/N < 7$. □

Next, we focus on the gain of network coding for broadcast sessions in wireless networks. Examples for wireless networks are provided in [24] and [25] in which the benefit of network coding in terms of reducing the number of transmissions (energy saving) of a broadcast session achieve the factors of 2 and $4/3$, respectively. Here, we prove that the gain network coding on the energy saving is bounded by 3.

Corollary 4.4. *The gain of network coding in terms of reducing the number of transmissions of a given broadcast session is bounded by a factor of 3.*

Proof of Corollary 4.4. Since the session is broadcast, $\mathcal{C}$ is a dominating set for the network graph $\mathcal{G}$. It is easy to show that a connected dominating set $\mathcal{U}$ can be constructed by adding at most $2|\mathcal{C}| - 2$ nodes to $\mathcal{C}$ (see [38]). A flow scheme that uses this connected dominating set can transport a broadcast packet to all nodes by at most $3|\mathcal{C}| - 2$ transmissions. From Lemma 4.1, we can conclude that the gain of network coding is bounded by a factor of $(3|\mathcal{C}| - 2)/|\mathcal{C}| < 3$. □

Note that the above bounds are tighter than the bounds which are presented in our earlier work [39].

Finally, we want to mention that there is a related work in [40] which shows the gain of network coding on broadcasting in large scale homogeneous networks is asymptotically a factor of $\Theta(\log(n))$. The paper assumes a probabilistic channel model (like epidemic model) for a correct reception in wireless radio range. Therefore, considering any arbitrary cutset, on average only a fraction of information sent through the cutset is usually decodable by the receivers. In such a network setting network coding can largely benefit, because, by combining the broadcast packets and sending them through a cutset, the receivers are able to decode all packets when enough number of coded packets are received correctly. This network coding scheme will be much more efficient than the simple routing scheme in which broadcast packets are set repeatedly till all nodes receive the packets correctly. However, in our work, we do not observe such large benefit from network coding, because, we assume a deterministic channel model.

There is also another network setting in which the gain of network coding on broadcasting has been widely studied. In this setting, which is called "index coding" problem, a source node intends to send some packets to its neighbors over a broadcast channel under the assumption that each neighbor already has some of the packets. Then, using coding techniques the source finds some proper combinations of its packets and broadcast them to all neighbors. Each neighbor decodes its designated packets, using the received coded packets and its already existed packets. It has been shown that the gain of index coding can be unboundedly large [41], [42]. However, notice that the index coding problem setting is quite different than ours, because, we do not assume that the nodes already have some side information about the source packets.

4.2 Throughput Gain of Network Coding in Wireless Networks

The throughput of a single multicast sessions in a directed graph has been a popular case of study in network coding papers [1], [8], [9], [10]. In [1], an example for a wired network with directed links is depicted where the gain of network coding on the throughput and energy is proportional to $\log(|\mathcal{V}|)$.

In Theorem 4.5, we prove that network coding can increase the throughput of a single multicast session in wireless networks by at most a constant factor, in contrast to wired networks where the throughput can be increased without bound by using network codes [1].

We emphasize that the results of [17] and [18] were obtained for wired network with bidirectional links and are not applicable for wireless network model. There are two main differences between the wired and wireless network models which make the network coding gain very different in these networks.

First, in wireless networks under Gaussian channel models we assume that a node receives one signal at a time (simple point-to-point coding for wireless communication). So, the receiving rate of every node is bounded by the wireless channel capacity. However, in the wired networks a node can receive several different bits over several links simultaneously, and there is no such bound for the receiving rate.

Second, under realistic wireless channel models as the ones we employ in this work, noise and interference constitute additional limiting factors for the network capacity. The SINR model for noise and interference—which forms an integral part of one of our channel models—leads to geometric constraints which in turn imply the fundamental bounds we derive. However, wired network models are not limited by such geometric constraints and can be in the form of any arbitrary directed or undirected graph.

Theorem 4.5. *Consider a single multicast session in an arbitrary wireless network. Then, the network coding gain on the throughput of the single multicast session is at most a constant factor $\sigma = \lceil 2 + (1+\Delta)\sqrt{d+3}\rceil^d$ under the protocol model and*

$$\sigma = (5^d - 2^d)\left\lceil \sqrt{d}\left(2 + \left(\frac{\beta \sum_{J\in\mathbb{Z}^d}^{|J|>0} |J|^{-\alpha}}{1-\rho^{-\alpha}}\right)^{\frac{1}{\alpha}}\right)\right\rceil^d$$

under the physical model, where $\rho > 1$ may be any constant such that the geometric graph model $\mathcal{G}$ of the wireless network stays connected for transmission range of $r_G = r_{\max}/\rho$.

Proof of Theorem 4.5. Consider an arbitrary terminal of the multicast session. The maximum rate at which data can be received by the terminal is equal to the wireless channel capacity W. By considering the cutset which separates the terminal from other nodes, we conclude that W is an upper bound on the throughput of a multicast session under any arbitrary network coding.

On the other hand, in [43] and [44] some flow schemes for the protocol model and the physical model are constructed with a broadcast throughput of W/σ for any connected wireless network, where σ is some constant which can be determined by the channel model and turns out to be the above value. Such flow schemes can be used to send the data from the source to all terminals with rate W/σ. This shows that network coding can improve the throughput of a single multicast session by a factor that is bounded by σ. □

Note that despite its apparent similarity to the proofs of previous work [43], [44], our argument is in fact different since network coding has not been taken into account in the mentioned existing work. Here, we study the gain of network coding on the throughput of a multicast session.

5 Bounds on the Gain of Network Coding for Multiple Unicast Sessions

In this section, we study the benefit of network coding for multiple unicast sessions in terms of energy and throughput. By assumption, energy savings are proportional to savings in the number of transmissions needed. We first bound the gain of network coding in terms of the number of transmissions, then in terms of throughput. In addition, we focus on value-blind network coding schemes and present novel criteria for evaluating the potential gain of network coding in wired or wireless networks.

5.1 Energy Gain of Network Coding

Here, we investigate the benefit of network coding in terms of energy saving for multiple unicast sessions. Note that in the single multicast scenario, network coding benefits by distributing the same information on different links and by efficiently using the links for sending the information toward several terminals. However, in multiple unicasts scenario, the links carry independent information of different unicast sessions. Intuitively, there is less chance for network coding to benefit in terms of the number of transmissions for unicast sessions. We will explore this fact more precisely later.

Theorem 5.1 provides a lower bound in terms of the number of transmissions for multiple unicast sessions. The lower bound is determined simply by considering the hop-count distance of the source and terminal nodes. Interestingly, the theorem is also valid for wired networks under the assumptions of Section 3.6.

Notably, the bounds of Theorem 5.1 apply to any set of (fully compressed) packets that are unicast from some sources to some terminals. This bound is tight when the set of sources and the set of terminals are distant from each other.

Theorem 5.1. *Assume that $b_1, b_2, \ldots, b_k$ are packets of some unicast sessions. Denote N as the average number of transmissions for transporting these packets in a given network. Also, denote the source and terminal of packet b_j by A_j and B_j for $j = 1, \ldots, k$. Then, under any arbitrary network coding scheme*

$$N \geq \max\left[\sum_{j=1}^{k} \ell(A_j, \mathcal{B}), \sum_{j=1}^{k} \ell(\mathcal{A}, B_j)\right]. \tag{6}$$

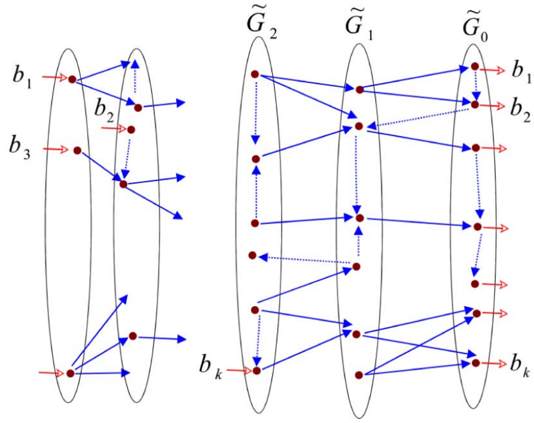

Fig. 3. Grouping the network nodes in terms of their distances from the set of terminals.

Remark. Note that in the sum of (6) the source and the terminal of a unicast session are repeated for each of the packets $b_1, b_2, \ldots, b_k$. Also, in the definition of N, notice that with *average* we mean the average number of transmissions per input packet over a large time interval (see Section 3.6).

Proof of Theorem 5.1. We group the network nodes in terms of their distances from the set of terminals ($\mathcal{B}$). We define $\tilde{\mathcal{H}}_0 = \mathcal{B}$ and $\tilde{\mathcal{H}}_i = \{u \in V : \ell(u, \mathcal{B}) \leq i\}$. Also, we define $\tilde{\mathcal{G}}_0 = \tilde{\mathcal{H}}_0$ and $\tilde{\mathcal{G}}_i = \tilde{\mathcal{H}}_i \backslash \tilde{\mathcal{H}}_{i-1}$. Note that the nodes of $\tilde{\mathcal{G}}_i$ are only connected to the nodes of $\tilde{\mathcal{G}}_{i-1}$, $\tilde{\mathcal{G}}_i$, and $\tilde{\mathcal{G}}_{i+1}$. Fig. 3 depicts the transmissions which carry the packets over $\tilde{\mathcal{G}}_0, \tilde{\mathcal{G}}_1, \ldots$.

Denote the number of sources which are not in $\tilde{\mathcal{H}}_i$ by q_i. Clearly, $q_i = \sum_j \mathbb{I}_{[\ell(A_j,\mathcal{B})>i]}$. Note that the cutset between $\mathcal{V}\backslash\tilde{\mathcal{H}}_i$ to $\tilde{\mathcal{H}}_i$ is the set of edges between $\tilde{\mathcal{G}}_{i+1}$ and $\tilde{\mathcal{G}}_i$ (in the directed graph, consider the edges directed from $\tilde{\mathcal{G}}_{i+1}$ to $\tilde{\mathcal{G}}_i$).

From the assumption the packets $b_1, b_2, \ldots, b_k$ are fully compressed, hence, at least q_i transmissions are needed for sending them from $\mathcal{V}\backslash\tilde{\mathcal{H}}_i$ to $\tilde{\mathcal{H}}_i$. Therefore, the total number of transmissions for transporting the packets is at least $\sum_{i=0}^{\infty} q_i$. Thus,

$$\begin{aligned} N &\geq \sum_{i=0}^{\infty} q_i = \sum_{i=0}^{\infty} \sum_{j=1}^{k} \mathbb{I}_{[\ell(A_j,\mathcal{B})>i]} \\ &= \sum_{j=1}^{k} \sum_{i=0}^{\infty} \mathbb{I}_{[\ell(A_j,\mathcal{B})>i]} \\ &= \sum_{j=1}^{k} \ell(A_j, \mathcal{B}). \end{aligned}$$

Similarly, by grouping the nodes in terms of their distances from the set of sources ($\mathcal{A}$) and applying the same method, we can show that $N \geq \sum_{j=1}^{k} \ell(\mathcal{A}, B_j)$. □

Applying this result to certain settings of particular interest leads to the striking conclusion that network coding should not be employed at all in these settings. In the first setting, all of the independent unicast connections have *the same source*, in the second setting, all connections have *the same terminal*. Surprisingly, network coding provides *no benefit* at all in terms of the number of transmissions in both cases (see Corollary 5.2).

Examples of such scenarios include *sensor networks* where the sensors send independent sensing information to a common sink (terminal). Clearly, they create a traffic pattern similar to case 1 of Corollary 5.2 below. Notice that Liu et al. [35] study this scenario more generally by considering a cost function in a single sink sensor networks. That paper establishes the following result: when the sources are independent and the cost is proportional to the number of transmissions, then shortest path routing protocol has the minimum cost. This agrees with our result for this special case; our proof, however, follows a different technique.

Another concrete example concerns a certain part of traffic in mesh networks with an *internet gateway*, namely, independent uplink traffic toward gateway (case 1) and independent downlink traffic from the gateway (case 2).

As follows from our main result, in mesh networks, energy consumption cannot be reduced using network coding as long as uplink and downlink traffic are treated separately. However, note that energy consumption would benefit from network coding when uplink and downlink packets are appropriately combined or in the case of multicast and broadcast [31], [32].

Corollary 5.2. *Consider multiple unicast sessions. For the following two scenarios, there is no network coding benefit in terms of the number of transmissions (energy).*

1. *A set of sources send their independent information to a single sink.*
2. *A single source sends independent information to each of a set of terminals.*

Proof of Corollary 5.2. We establish the claim by showing that the shortest path flow scheme is the most efficient scheme among all flow and network coding schemes. If we use the flow scheme that transports the packets over the shortest path, then the number of transmissions is equal to $\sum_{i=1}^{k} \ell(A_i, \mathcal{B})$ in case 1 and equal to $\sum_{i=1}^{k} \ell(\mathcal{A}, B_i)$ in case 2 of Theorem 5.1. This means that network coding does not benefit in terms of reducing the number of transmissions for these scenarios. □

5.2 Throughput Gain of Network Coding in Wireless Networks

Here, we bound the gain of network coding on the throughput of multiple unicast sessions in wireless networks. We study the transport capacity and the throughput of the sessions under an arbitrary network coding.

Theorem 5.3 shows that the maximum transport capacity of an arbitrary network under an arbitrary network coding scheme is bounded by the constant factor π of the maximum transport capacity computed under flow schemes. In other words, the gain of network coding on the maximum transport capacity of arbitrary wireless network is bounded by a factor of π.

Moreover, this result shows that the upper bounds on the transport capacity computed in previous works [2], [3], [4], [5] increase by at most factor of π if we employ network coding in a wireless network.

Theorem 5.3. *Denote the maximum transport capacity of an arbitrary wireless network by using flow schemes and network coding schemes by C_T^{f} and C_T^{mc}, respectively. Then,*

$$C_T^{\mathrm{mc}} \leq K_d \cdot C_T^{\mathrm{f}}, \tag{7}$$

where $K_d = 2$ if $d = 1$ and $K_d = \pi$ if $d = 2, 3$ (d is the dimension).

Proof of Theorem 5.3. We prove the theorem in $d = 2$ dimensional space. The proof can be easily extended for $d = 1, 3$ dimensional space using the same method.

First, we prove the following lemma to capture certain geometric properties of wireless networks. □

Lemma 5.4. *Consider an arbitrary set of vectors $\mathcal{Q} = \{\vec{a_1}, \ldots, \vec{a_k}\}$ in d-dimensional space. Then, there exists a unit vector $\vec{i}$ and a set $\underline{\mathcal{Q}} \subseteq \mathcal{Q}$ such that*

$$\sum_{\vec{a_j} \in \mathcal{Q}} |\vec{a_j}| \leq K_d \sum_{\vec{a_j} \in \underline{\mathcal{Q}}} \vec{a_j} \cdot \vec{i} \tag{8}$$

(K_d is the same as Theorem 5.3).

Proof of Lemma 5.4. First we establish the claim for the case $d = 2$, i.e., for a 2D space. Denote by $\vec{i_\theta}$ the unit vector which includes the angle θ with the horizontal axis. Also, denote the angles of vectors $\vec{a_1}, \ldots, \vec{a_k}$ with the horizontal axis by $\varphi_1, \ldots, \varphi_k$.

By summing over $\vec{a_j}$ and integrating over θ, we find

$$\begin{aligned}\int_0^{2\pi} \sum_{\vec{a_j} \in \mathcal{Q}} |\vec{i_\theta} \cdot \vec{a_j}| d\theta &= \sum_{\vec{a_j} \in \mathcal{Q}} \int_0^{2\pi} |\vec{a_j}| \cdot |\cos(\theta - \varphi_j)| d\theta \\ &= \sum_{\vec{a_j} \in \mathcal{Q}} |\vec{a_j}| \int_0^{2\pi} |\cos\theta| d\theta \\ &= 4 \sum_{\vec{a_j} \in \mathcal{Q}} |\vec{a_j}|.\end{aligned}$$

By the *mean value theorem of integration*, there exists θ_0 such that

$$2\pi \sum_{\vec{a_j} \in \mathcal{Q}} |\vec{i}_{\theta_0} \cdot \vec{a_j}| = 4 \sum_{\vec{a_j} \in \mathcal{Q}} |\vec{a_j}|. \tag{9}$$

Next, we partition $\mathcal{Q}$ into two sets: $\mathcal{Q}_1 = \{\vec{a_j} \in Q : \vec{i}_{\theta_0} \cdot \vec{a_j} \geq 0\}$ and $\mathcal{Q}_2 = \mathcal{Q} \setminus \mathcal{Q}_1$. Then,

$$\sum_{\vec{a_j} \in \mathcal{Q}} |\vec{i}_{\theta_0} \cdot \vec{a_j}| = \sum_{\vec{a_j} \in \mathcal{Q}_1} \vec{i}_{\theta_0} \cdot \vec{a_j} + \sum_{\vec{a_j} \in \mathcal{Q}_2} \left(- \vec{i}_{\theta_0} \right) \cdot \vec{a_j}. \tag{10}$$

By (9) and (10), we conclude one of the pairs $(\vec{i}_{\theta_0}, Q_1)$ and $(-\vec{i}_{\theta_0}, \mathcal{Q}_2)$ can be considered as $(\vec{i}, \underline{\mathcal{Q}})$ to satisfy (8).

For the case $d = 1$, we set $\theta_0 = 0$ and define $\mathcal{Q}_1$ and $\mathcal{Q}_2$ similarly. It follows easily that (8) holds with $K_1 = 2$.

For the case $d = 3$, we apply spherical coordinates. We write a unit vector $\vec{k}$ as the sum of two orthogonal vectors such that one of them is parallel to $\vec{a_j}$. Without

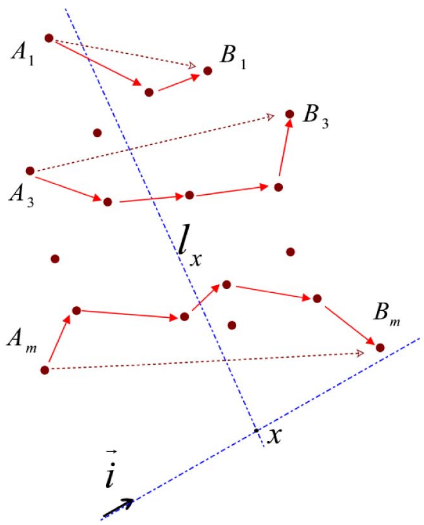

Fig. 4. l_x is an geometric cutset for some source-terminal pairs of $\underline{\mathcal{AB}}$.

lack of generality, assume that $\vec{a_j}$ is in the direction of the x-axis and that $\vec{k} = (1, \theta, \phi)$. Then

$$\int_{|\vec{k}|=1} |\vec{k} \cdot \vec{a_j}| = |\vec{a_j}| \int_0^{\pi} \int_0^{2\pi} |\cos(\theta)| d\theta \sin(\phi) d\phi = 8|\vec{a_j}|.$$

On the other hand, $\int_{|\vec{k}|=1} 1 = 4\pi$. Again, from the *mean value theorem* it follows that there exists a unit vector $\vec{k_0}$ such that $4\pi \sum_{\vec{a_j}\in\mathcal{Q}} |\vec{k_0} \cdot \vec{a_j}| = 8 \sum_{\vec{a_j}\in\mathcal{Q}} |\vec{a_j}|$. The rest of proof follows along similar lines as before, thereby constructing $(\vec{k_0}, \mathcal{Q}_1)$ and $(-\vec{k_0}, \mathcal{Q}_2)$. □

Now, consider an arbitrary set of unicast sessions $\mathcal{AB} = \{(A_1, B_1), \dots, (A_k, B_k)\}$ with average rates $R_1, \dots, R_k$. We define $\vec{a_j} = R_j \cdot \overrightarrow{A_j B_j}$. By Lemma 5.4, there exists a unit vector $\vec{i}$ and $\underline{\mathcal{AB}} \subseteq \mathcal{AB}$ such that

$$\sum_{(A_j,B_j)\in\mathcal{AB}} R_j \left|\overrightarrow{A_j B_j}\right| \le K_d \sum_{(A_j,B_j)\in\underline{\mathcal{AB}}} R_j \overrightarrow{A_j B_j} \cdot \vec{i}. \tag{11}$$

Next, we rotate the Cartesian axes such that the axis X is aligned on the direction of unit vector $\vec{i}$. We denote the orthogonal line which crosses axis X at point x by l_x (see Fig. 4). Also, we denote $\mathbb{I}_{[A_j B_j \to l_x]}$ as the indicator function that the line segment $A_j B_j$ has been intersected by line l_x. Note that if $(A_j, B_j) \in \underline{\mathcal{AB}}$ and $\mathbb{I}_{[A_j B_j \to l_x]} = 1$, then we can show that A_j is located in the left side and B_j is located in the right side of l_x because $\overrightarrow{A_j B_j} \cdot \vec{i} > 0$ (see the proof of Lemma 5.4). Therefore, $\sum_{(A_j,B_j)\in\underline{\mathcal{AB}}} R_j \mathbb{I}_{[A_j B_j \to l_x]}$ is the average rate of information for the unicast sessions of $\underline{\mathcal{AB}}$ which go from the left to the right side of line l_x.

Denote the set of simultaneous transmissions at time instant τ by $\mathcal{SD}_\tau = \{(S_1, D_1), \dots, (S_m, D_m)\}$. Then, the rate of information is transmitted across l_x at this time instant is $\sum_j W_j(\tau) \mathbb{I}_{[S_j D_j \to l_x]}$. Therefore, from the definitions, we have

$$\sum_{(A_j,B_j)\in\underline{\mathcal{AB}}} R_j \mathbb{I}_{[A_j B_j \to l_x]} \le \lim_{T\to\infty} \frac{1}{T} \int_0^T \sum_{(S_j,D_j)\in\mathcal{SD}_\tau} W_j(\tau) \mathbb{I}_{[S_j D_j \to l_x]} d\tau. \tag{12}$$

Next, we integrate over all possible positions of l_x:

$$\begin{aligned}
&\sum_{(A_j,B_j)\in\underline{\mathcal{AB}}} R_j \overrightarrow{A_j B_j} \cdot \vec{i} \\
&= \sum_{(A_j,B_j)\in\underline{\mathcal{AB}}} R_j \int_{-\infty}^{\infty} \mathbb{I}_{[(A_j,B_j)\to l_x]} dx \\
&= \int_{-\infty}^{\infty} \sum_{(A_j,B_j)\in\underline{\mathcal{AB}}} R_j \mathbb{I}_{[(A_j,B_j)\to l_x]} dx \\
&\le \int_{-\infty}^{\infty} \lim_{T\to\infty} \frac{1}{T} \int_0^T \sum_{(S_j,D_j)\in\mathcal{SD}_\tau} W_j(\tau) \mathbb{I}_{[S_j D_j \to l_x]} d\tau \, dx \\
&= \lim_{T\to\infty} \frac{1}{T} \int_0^T \sum_{(S_j,D_j)\in\mathcal{SD}_\tau} W_j(\tau) \int_{-\infty}^{\infty} \mathbb{I}_{[S_j D_j \to l_x]} dx \, d\tau \\
&= \lim_{T\to\infty} \frac{1}{T} \int_0^T \sum_{(S_j,D_j)\in\mathcal{SD}_\tau} W_j(\tau) \left|\overrightarrow{S_j D_j} \cdot \vec{i}\right| d\tau \\
&\le \lim_{T\to\infty} \frac{1}{T} \int_0^T \sum_{(S_j,D_j)\in\mathcal{SD}_\tau} W_j(\tau) \left|\overrightarrow{S_j D_j}\right| d\tau \\
&\le \max_{\mathcal{SD},\tau} \sum_{(S_j,D_j)\in\mathcal{SD}_\tau} W_j(\tau) \left|\overrightarrow{S_j D_j}\right| = C_T^{\mathrm{f}}.
\end{aligned}$$

Finally, by (11) we conclude that

$$C_T^{\mathrm{mc}} = \max_{\mathcal{AB}} \left(\sum_{(A_j,B_j)\in\mathcal{AB}} R_j \left|\overrightarrow{A_j B_j}\right| \right) \le K_d C_T^{\mathrm{f}}. \tag{13}$$

Next, we study the gain of network coding on the throughput of large homogeneous networks which is a popular model in network capacity papers [2], [3], [4], [45], [46]. Corollary 5.5 proves that network coding does not change the asymptotic behavior of the throughput of large homogeneous wireless networks.

Prior work (see [6]) shows a similar result under the protocol model for the wireless channel. Here, we establish the bound also under other channel models, i.e., the nonsymmetric protocol model [2], [3], the physical model, and the generalized physical models [4]). Further, our technique of proof is different from [6] since we base our argument on the transport capacity.

Corollary 5.5. *Network coding gain on the throughput of large homogeneous wireless networks is bounded by a constant factor.*

Proof of Corollary 5.5. Previous work (see [2], [3], [4]) provides some asymptotic upper bounds on the throughput (sum of the rates of the unicast sessions) of large homogeneous networks under various channel models. This is done in two steps. First, an upper bound on the transport capacity (C_T^{f}) is computed. Second, the transport capacity is divided by the average distance of source-terminal pairs (L). On the other hand, [2], [3], [45], [46] provide some flow schemes to achieve a throughput within a constant factor of the computed upper bounds. The throughput of these schemes represents tight lower bounds on the throughput capacity of the network.

Now, by Theorem 5.3, the gain of network coding on the transport capacity is bounded by a factor of K_d. Therefore, if we employ network coding then the throughput will be smaller than K_d multiply by the traditional upper bounds [2], [3], [4] (which have been computed for flow schemes). We also note that the flow schemes can achieve the traditional upper bounds up to a constant factor [2], [3], [45], [46]. From these, we conclude that the network coding gain on the throughput is bounded by a constant. □

5.3 Energy and Throughput Gain of Value-Blind Network Coding Schemes

In this section, we study the potential gain of network coding for a wide class of network codes, namely, *value-blind* schemes. We call a network coding *value-blind* if for every intermediate node the size of the output packet is only a function of the size of the input packet (i.e., the number of bits in an input packet), but not a function of the values of the bits, consequently, the number of transmissions used in the network coding scheme stays the same when the values of bits of the inputs varies. As an example, when the sources optimally code their messages, then no coding gain can be achieved from looking at the actual contents of the packets. Thus, optimally source coded traffic always leads to value-blind network coding. In this sense, the assumption of being value-blind is not very restrictive. Indeed, all commonly used network codes are value-blind [1], [8], [9], [10], [16].

Our main tools, Theorems 5.6 and 5.10 relate an arbitrary value-blind network coding scheme to a flow scheme. These theorems can be used as criteria for evaluating the potential gain of network coding on multiple unicast sessions in a given wired or wireless network. Putting these theorems to work, we identify several scenarios where network coding *cannot* improve the network performance in terms of throughput or number of transmissions (see Corollaries 5.7 to 5.11).

Theorem 5.6. *For any value-blind network coding there exists a permutation of the destinations and a flow scheme which transports the same packets from the sources to that permutation of the destinations using a subset of the transmissions of the coding network scheme.*

Proof of Theorem 5.6. Let us start the proof with a more detailed statement.

Assume that a value-blind network coding scheme transports packets $b_1, \ldots, b_k$. Denote the source and destination of packet b_j by A_j and B_j for $j = 1, 2, \ldots, k$ (a node can be the source or destination for several packets). Then, there exists a permutation $\sigma(\cdot)$ and a flow scheme from A_j to $B_{\sigma(j)}$ for $j = 1, 2, \ldots, k$ that can transport the same packets using a subset of the transmissions used by the network coding scheme; equivalently, the flow scheme can transport the packets from $A_{\sigma'(j)}$ to B_j for $j = 1, 2, \ldots, k$ where $\sigma'(\cdot)$ is the inverse of $\sigma(\cdot)$.

We prove the statement for wired and wireless networks separately, since the graph models of these two are different.

1. *Wired networks.* We add a virtual *supersource* node A_0 to the set of nodes. We assume that A_0 generates the packets $b_1, \ldots, b_k$ and sends them through k different links to $A_1, \ldots, A_k$ (each link transports one packet). Similarly, we add a virtual *superdestination* node B_0 and assume that it receives the packets from $B_1, \ldots, B_k$ through different links. In other words, we assume that all packets are generated at A_0 and then transported to B_0.

   We construct a directed graph $\bar{\mathcal{G}} = (\bar{\mathcal{V}}, \bar{\mathcal{E}})$ by considering the set of transmissions which transport the packets in the wired network. We set $\bar{\mathcal{V}} = \mathcal{V} \cup \{A_0, B_0\}$. For every transmission (a transmission transports one packet), we consider a directed edge between its sender and its receiver. Note that there would exist more than one edge between two nodes of $\bar{\mathcal{G}}$. The graph $\bar{\mathcal{G}}$ depicts how the packets are generated, combined, and transported at different stages from A_0 to B_0 in the network.

   Now, we claim that the size of a minimum cutset directed from A_0 to B_0 in graph $\bar{\mathcal{G}}$ is larger than or equal to k. For a proof, we consider an arbitrary cutset between A_0 and B_0. Note that each edge of the cutset corresponds to the transmission of one packet of combined information. By assumption, the packets $b_1, \ldots, b_k$ are fully compressed and the network coding scheme is value-blind (i.e., the transmissions have no side information about the transported packets), therefore, at least k transmissions (directed edges on the cutset) are needed to losslessly transport the packets from the part which contains A_0 to the part which contains B_0.

   Next, we apply Menger's theorem for edge-disjoint paths (also called min-cut max-flow theorem) [47] and use the above claim. The theorem shows that there exist at least k edge-disjoint paths from A_0 and B_0 in $\bar{\mathcal{G}}$. From the construction of $\bar{\mathcal{G}}$, we can conclude that these k disjoint paths connect A_j to $B_{\sigma(j)}$ for $j = 1, 2, \ldots, k$ where $\sigma(\cdot)$ is a permutation of $(1, \ldots, k)$. We construct the flow scheme by considering these paths which can be used for transporting the packets from A_j to $B_{\sigma(j)} (j = 1, 2, \ldots, k)$.

2. *Wireless networks.* In wireless networks, the broadcast nature of the wireless channel possesses a different graph model. Here, when a node transmits several neighbor nodes can receive the packet simultaneously. We construct a directed graph $\hat{\mathcal{G}} = (\hat{\mathcal{V}}, \hat{\mathcal{E}})$ based on this property of the network.

   First, we consider two nodes A_0 and B_0 in $\hat{\mathcal{V}}$ similar to the proof of wired network case. Next, we repeat every node of $\mathcal{V}$ in $\hat{\mathcal{V}}$ equal to the number of times that it transmits. For example, assume that $v \in \mathcal{V}$ transmits s times, then we consider $v^{(1)}, \ldots, v^{(s)}$ in $\hat{\mathcal{V}}$ which correspond to the transmissions of v ordered by the time of transmission. To be more precise on how to construct the graph $\hat{\mathcal{G}}$, we arrange the nodes of $\hat{\mathcal{V}}$ from left to right based on their transmission times. We put an edge from $v^{(i)}$ to $u^{(j)}$ if for some $i' \geq i$, the transmission of $v^{(i')}$ is received by the node u before the time that the transmission of $u^{(j)}$ occurs. Also, we put edges from $v^{(i)}$ to $v^{(j)}$ for all $v \in \mathcal{V}$ and $i < j$. The graph $\hat{\mathcal{G}}$ depicts how the packets are combined and transported from the far left node A_0 to the far right node B_0 by different transmissions over time.

Now, we claim that the size of a minimum disconnecting-vertex-set between A_0 and B_0 in $\hat{\mathcal{G}}$ is larger than or equal to k. A disconnecting-vertex-set is a set of nodes with the following property: if they are removed (with their edges), then there will exist no path from A_0 to B_0 in $\hat{\mathcal{G}}$. The proof of the claim is similar to our proof for the wired network case. We note that a disconnecting-vertex-set forwards the packets from the left part (which contains A_0) to the right part (which contains B_0) in $\hat{\mathcal{G}}$. Also, each node in $\hat{\mathcal{V}}$ corresponds to one transmission, i.e., it carries one packet and it carries no side information about the packets in the value-blind network coding scheme. Therefore, at least k transmissions are needed for losslessly transporting the packets from the part which contains A_0 to the part which contains B_0. This implies that the size of minimum disconnecting-vertex-set is at least k.

Then we apply Menger's theorem for vertex-disjoint paths [47] and use the above claim. It shows that there exist at least k vertex-disjoint paths from A_0 and B_0 in $\hat{\mathcal{G}}$. From the construction of $\hat{\mathcal{G}}$, we can conclude that a set of transmissions which correspond the internal vertices of these paths can transport the packets from A_j to $B_{\sigma(j)}$ for $j = 1, 2, \dots, k$ where $\sigma(\cdot)$ is a permutation of $(1, 2, \dots, k)$. □

In Corollary 5.7, we compute a lower bound for the energy consumption of value-blind network coding schemes that is tighter than the lower bound given in Theorem 5.1. Moreover, in Corollary 5.8, we identify new scenarios where network coding does not provide any benefit in terms of energy savings.

Corollary 5.7. *In the notation of the proof of Theorem 5.6, denote the number of transmissions used in an arbitrary value-blind network coding scheme for transporting the packets by N. Then,*

$$N \geq \min_{\sigma} \left(\sum_{j=1}^{k} \ell(A_j, B_{\sigma(j)}) \right). \tag{14}$$

Proof of Corollary 5.7. We consider an arbitrary value-blind network coding scheme and apply Theorem 5.6. We find the corresponding permutation $\sigma(\cdot)$ and the flow scheme. The number of transmissions used by the flow scheme is at least $\sum_{j=1}^{k} \ell(A_j, B_{\sigma(j)})$. Clearly, this is also a lower bound on N. Note that (14) takes the minimum of $\sum_{j=1}^{k} \ell(A_j, B_{\sigma(j)})$ over all permutations $\sigma(\cdot)$, therefore, it gives us a general lower bound on the number of transmissions (N) of any arbitrary value-blind network coding scheme. □

Corollary 5.8. *In the notation of the proof of Theorem 5.6, assume that we have $\sum_{j=1}^{k} \ell(A_j, B_j) = \min_{\sigma}(\sum_{j=1}^{k} \ell(A_j, B_{\sigma(j)}))$. Then, the flow scheme via shortest path is the most efficient in terms of number of transmissions among all value-blind network coding schemes.*

Proof of Corollary 5.8. From Corollary 5.7, the number of transmissions used by any value-blind network coding scheme is at least $\min_{\sigma}(\sum_{j=1}^{k} \ell(A_j, B_{\sigma(j)}))$. Since, the flow scheme achieves this lower bound, so no network coding scheme can be more efficient. □

In Corollary 5.9, we introduce some network scenarios in wired/wireless networks where network coding has no benefit in terms of either throughput or energy saving in the network.

Corollary 5.9 covers the scenarios discussed in Corollary 5.2 as it deals with scenarios in which the sources or destinations of independent unicast sessions are allowed to be rearranged arbitrarily. As we show, in such a scenario any given value-blind network coding scheme can be replaced by a flow scheme with the same or a better performance. Let us mention two such scenarios.

We call the first scenario *multiple-sink reporting*. In this scenario, a set of sources send (report) their independent information to a set of sinks, however the information can be sent to any sink (no matter which one). This scenario often occurs in sensor networks, where the sensors act as sources sending independent information to the sinks.

We call the second scenario *multiple-storage downloading*. In this scenario, a set of terminals receive (download) independent unicast information from a set of sources (storages), however every source has access to the entire information which is requested by the terminals. Such scenario occurs for Internet sites that provide file downloading services to independent clients using multiple data storages.

Corollary 5.9 (Scenarios of No Network Coding Gain). *Any value-blind network coding scheme that transports information in a "multiple-sink reporting" or "multiple-storages downloading" scenario can be replaced by a flow scheme with the same or better throughput and energy saving.*

Proof of Corollary 5.9. Similarly to the notation of the proof of Theorem 5.6, denote the source and destination of the transported packets using the network coding scheme by $A_1, \dots, A_k$ and $B_1, \dots, B_k$. Theorem 5.6 proves that there exists a flow scheme which transport the packets from $A_1, \dots, A_k$ to $B_{\sigma(1)}, \dots, B_{\sigma(k)}$. Since, for "multiple-sink reporting" scenario the destinations can be rearranged in arbitrary way, this the flow scheme can be employed for transporting the packets.

For "multiple-storage downloading" scenario, we consider the flow scheme which transports the packets from $A_{\sigma'(1)}, \dots, A_{\sigma'(k)}$ to $B_1, \dots, B_k$. Since, the information packets are available in all sources, we can use the flow scheme for transporting the packets to their corresponding terminals.

Since the mentioned flow scheme uses only a subset of the transmissions used by the network coding scheme, it provides the same or better throughput and energy saving than the network coding scheme. □

In Theorem 5.10, we relate a value-blind network coding scheme to a flow scheme for the scenarios where the average rates of the unicast sessions are given. Using this theorem, we identify some scenarios where network coding does not provide any gain on the throughput (see Corollary 5.11). Note that the throughput of a multiple unicast sessions is defined and compared based on the average rate vector of the unicast sessions $\mathbf{R} = (R_1, \ldots, R_k)$. Therefore, in Corollary 5.11, we consider in general an arbitrary partial ordering relation ($\leq^o$) defined for $\mathbf{R} \in \mathbb{R}^k$ to find more scenarios where network coding has no gain on the throughput.

Theorem 5.10. *Assume that a value-blind network coding scheme transports information from source A_j to destination B_j at average rate R_j (packets per second) for $j = 1, 2, \ldots, k$. Then, there exists a flow scheme which transports information from the set of sources $\{A_1, \ldots, A_k\}$ to the set of destinations $\{B_1, \ldots, B_k\}$ such that the average sending rate of A_j and receiving rate of B_j are equal to R_j for all j, and it uses a subset of the transmissions used by the network coding scheme.*

Remark. Note that the flow scheme guarantees the transport of the packets between *sets* of sources and destinations but will not necessarily match the sources and destinations of the network coding scheme. Still, such an application might become useful in certain scenarios.

Proof of Theorem 5.10. We consider the transported packets by the network coding scheme in the time period $[0, T]$ where $T \to \infty$. The scheme transports $n_j \simeq \lfloor R_j T \rfloor$ packets from A_j to B_j for all $j = 1, 2, \ldots, k$. We repeat the nodes A_j and B_j each n_j times for all j, so that a pair of source and destination is assigned to every transported packet. Then, we apply Theorem 5.6. The theorem proves there exists a flow scheme which transports $\sum_{j=1}^{k} n_j$ packets from the (repeated) sources $A_1, \ldots, A_k$ to a permutation of the (repeated) destinations $B_1, \ldots, B_k$. Since, A_j and B_j nodes are repeated n_j times as source and destination of the transported packets, we conclude that the average sending rate of A_j and receiving rate of B_j are equal to $R_j \simeq \frac{n_j}{T}$ for all j in the flow scheme. Therefore, the flow scheme satisfies the conditions of Theorem 5.10. □

Corollary 5.11. *Assume that a flow scheme $\mathcal{F}$ transports information from source A_j to destination B_j at average rate R_j for $j = 1, 2, \ldots, k$. Let $\leq^o$ denote an arbitrary partial ordering relation defined on $\mathbb{R}^k$. Also, assume that for any other flow scheme $\mathcal{F}'$ which transports information from $\{A_1, \ldots, A_k\}$ to $\{B_1, \ldots, B_k\}$ we have $\mathbf{R} \not\leq^o \mathbf{R}'$, where $\mathbf{R}' = (R'_1, \ldots, R'_k)$ and R'_j denotes the average sending rate of A_j and receiving rate of B_j for $j = 1, 2, \ldots, k$ under $\mathcal{F}'$.*

Then, there exists no value-blind network coding scheme with a higher throughput (based on $\leq^o$) than the flow scheme $\mathcal{F}$.

Proof of Corollary 5.11. From Theorem 5.10, for any value-blind network coding scheme which transports information from A_j to B_j at rate R'_j, there exists a flow scheme $\mathcal{F}'$ which transports the packets from $\{A_1, \ldots, A_k\}$ to $\{B_1, \ldots, B_k\}$ such that the average sending rate of A_j and receiving rate of B_j are R'_j. By the assumptions, we have $\mathbf{R} \not\leq^o \mathbf{R}'$. This shows that the rate vector of the network coding scheme ($\mathbf{R}'$) is not higher than of the rate vector of $\mathcal{F}$ based on $\leq^o$. □

We can define $\leq^o$ in several ways. For instance, we can consider the sum of the rates $\|\mathbf{R}\|_{L1} = \sum_j R_j$ to define the partial ordering relation. Then, the corollary demonstrates that the throughput of a flow scheme which has the maximum sum of the rates among all permutations of source and destination pairs, cannot be improved further using a value-blind network coding scheme.

Also, we may find the following partial ordering relation useful for some scenarios: $\mathbf{R} \leq^o \mathbf{R}'$ if $R_j \leq R'_j$ for all $j = 1, \ldots, k$. In this partial ordering relation, a scheme has a higher throughput than another scheme if and only if it provides a higher rate for all unicast sessions. For example, in the scenarios where the paths of unicast sessions intersect each other, increasing the rate of one session can decrease the rate of another session. In such scenarios, applying this partial ordering relation with Corollary 5.11 reveals more cases where the throughput cannot be improved further by employing value-blind network coding schemes.

However, we must notice that for some scenarios finding the most efficient routing scheme (in terms of power consumption or throughput), can be computationally very complex or require global knowledge on the network, while network coding schemes exist which use distributed and local algorithms to achieve almost the same performance. This means that even for scenarios wherein network coding cannot add to the performance compare to the best routing solution it would be worth to be employed. To be fair, we should point out that employing a centralized or distributed network coding scheme also requires some global or local knowledge on the network topology and computational complexity for the nodes that would be a large overhead as well.

6 Conclusion

In this work, we studied fundamental limitations of the benefit of network coding for arbitrary wireless multihop networks. We focused on two popular network scenarios: single multicast session and multiple unicast sessions.

We proved that the benefit of network coding in terms of throughput or energy saving is bounded by a constant factor for any single multicast session in any wireless networks. Also, we computed bounds for the gain of network coding in terms of the number of transmissions for multiple unicast sessions, in the sense that there exists a flow scheme that achieves the same performance as the network coding up to the claimed bound. We found that network coding provides no benefit in terms of energy savings in sensor networks where the sensors gather independent information for the sink or in mesh networks for unidirectional traffic from/toward the gateway. Moreover, we proved that network coding can increase the maximum transport capacity of an arbitrary wireless network by at most a factor of π. This implies that network coding does not change the throughput of large homogeneous networks by more than a constant.

Furthermore, we studied the potential gain of value-blind network coding schemes in wired or wireless networks. We presented theorems which relate a value-blind network coding scheme to a flow scheme that transports the packets from the sources to a permutations of destinations. We showed that for several network scenarios network coding provides no gain in terms of throughput or energy saving.

Finally, from the established bounds, complemented with previous related work on capacity bounds [2], [3], [4], [5], [6], we conclude that network coding plays valuable but not crucial role with regards to the performance of wireless networks. Rather, channel interference and topology of wireless networks seem to emerge as the determinant parameters on the throughput and energy consumption.

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