Bounds on the Benefit of Network Coding: Throughput and Energy Saving in Wireless Networks

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***Abstract*—In this paper we establish fundamental limitations to the benefit of network coding in terms of energy and throughput in multihop wireless networks. Thereby we adopt two well accepted scenarios in the field: single multicast session and multiple unicast sessions. Most of our results apply to arbitrary wireless network and are, in particular, not asymptotic in kind.**

In terms of *throughput and energy saving* we prove that the gain of network coding of a single multicast session is at most a constant factor. Also, we present a lower bound on the expected number of transmissions of multiple unicast sessions under an arbitrary network coding. We identify scenarios for which the network coding gain for energy saving becomes surprisingly close to 1, in some cases even exactly 1, corresponding to no benefit at all. Interestingly, we prove that the gain of network coding in terms of *transport capacity* is bounded by a constant factor π in any arbitrary wireless network and for all traditional channel models. This shows that the traditional bounds on the transport capacity [1]–[4] do not change more than constant factor π if we employ network coding. As a corollary, we find that the gain of network coding on the *throughput* of large scale homogeneous wireless networks is asymptotically bounded by a constant. Note that our result is more general than the previous work [5] and it is obtained by a different technique. In conclusion, we show that in contrast to wired networks, the network coding gain in wireless networks is constraint by fundamental limitations.

I. Introduction

In recent years, network coding has become an important research topic in network information theory. It has been shown that network coding can help improve the throughput and energy consumption of communication networks. Theoretical studies of network coding provide guidelines for designing or improving efficient and high performance wired or wireless networks. In this paper, we present theory leading to fundamental bounds on the gain network coding in arbitrary wireless networks. In particular, our work reveals that the benefit of network coding is limited by constants depending only on the dimension of the underlying space.

Today, studies on network coding have grown to large numbers, mostly focussing on benefits in terms of throughput or energy saving. In this paper we study the fundamental limitation of network coding in the same terms and identify scenarios where the network coding gain on the performance is noticeably small. To the best of our knowledge, this is the first paper which studies the bounds on the gain of network coding in terms of energy saving in wireless networks.

As the first contribution of this paper, we prove that the *energy gain* of network coding for a single multicast session is bounded by a small constant factor in an arbitrary wireless network. The constant factor is 17 for planar wireless networks. This lies in stark contrast to wired network where the network coding gain in terms of energy consumption and throughput can be unboundedly large [6].

As the second contribution, we bound the *throughput gain* of network coding for a single multicast session in an arbitrary wireless network. We show that network coding can improve the throughput in wireless networks by at most a constant factor which is determined by the parameters of the underlying wireless channel model.

As the third contribution, we study the *energy gain* for multiple unicast sessions in an arbitrary wireless network. We compute a novel lower bound on the expected number of transmissions. Assuming that each transmission carries the same size packet and that energy consumption is proportional to the number of transmissions, we find thus an upper bound on the energy gain of network coding. In addition, we identify important scenarios where network coding benefits in terms of energy consumption constitute a relatively small amount, in some cases *none at all*, e.g., for certain traffic patterns in wireless sensor and mesh networks.

As the fourth contribution, we study the benefit of network coding in terms of the *throughput* of wireless networks for multiple unicast sessions. We prove that network coding can increase the *transport capacity* by at most a constant factor π in an arbitrary wireless network. From this, we conclude that the traditional bounds on the transport capacity [1]–[4] would increase by at most a factor of π if we employed network coding. We also prove that the gain of network coding on the throughput of large homogeneous wireless networks is most a constant factor. This result is more general than the result of previous work [5] which has been proved for only a particular topology and wireless channel model.

In summary, we provide several important novel insights, especially on sensor and mesh networks, and generalize to arbitrary networks several results that had been known only for special cases.

The paper is organized as follows. In Section II, we review some related work. We introduce the network model and notations in Section III. We bound the network coding gain in terms of energy and throughput for a single multicast in Section IV. Next, in Section V, we study the benefit of network coding in wireless networks for multiple unicast sessions. Finally, we conclude the paper in Section VI.

II. Related Work

Network coding was first introduced in the seminal paper by Ahlswede et al. [7] in which it was proven that the maximum flow capacity of a single multicast session can be achieved using network coding in an arbitrary network with directional links. Later, [8] and [9] show constructively that the linear network codes can achieve the capacity of a single multicast session as well. Since then, a large body of work has explored the construction of efficient network coding algorithms, e.g., [6], [8]–[15].

For the networks with bidirectional links, [16], [17] show that network coding improves the throughput by at most a constant factor 2 for a single multicast. The constant factor turns out to be equal to one (no benefit) in the case of a single unicast or a broadcast. Note that these results do not extend for wireless networks where the channel is considered bidirectional. In fact, network coding combined with wireless broadcasting can potentially improve the performance in terms of throughput, energy efficiency, and congestion control in wireless networks [18], [19].

The potential of network coding for energy savings in broadcasting in ad hoc networks was established in [20], [21]. Here, we complement this valuable insight by show that such a gain is in fact bounded by a small constant for any wireless multicast. Moreover, we clearly strengthen the results of [22] for the maximum flow achievable in random wired and wireless networks (modeled as geometric random graphs) for a single multicast session by taking wireless interference into account.

The case of multiple unicast sessions in wireless networks is studied in [23], [24]. They studied cases where network coding would provide only marginal benefits. Most recent work [5] shows that the unicast capacity gain of network coding in ad hoc networks is asymptotically bounded by a constant factor, as the network is scaled to large size. We would like to take the opportunity to point out that we provide more general results which are non-asymptotical and valid for any network.

It should be mentioned that some papers assume correlation between the generated information from different sources in the network. They combine network coding with Distributed Source Coding (DSC) techniques to optimize the coding gain in the network [25], [26]. However, in this paper, we focus on pure network coding gain. In other words, we assume that the sources generate independent information.

III. Model Assumptions

In this section, we describe the models and notions used in this paper. A communication network is a collection of directed (or undirected) links connecting communication devices (nodes). The links can be established through actual wired or wireless transmissions. Here, we consider an arbitrary wireless networks. We assume that the channel is bidirectional and multiple-access in wireless network.

We emphasize that we consider an *arbitrary* topology for the wireless network. However, we assume that the topology has the *connectivity* which is needed for establishing the demanded sessions. We assume that the nodes are distributed in d-dimensional Euclidean space $\mathbb{R}^d$ and denote the set of nodes by V.

A. Wireless Channel Models

We employ the Protocol and the Physical Model [1], [2] for modeling the wireless channel. We denote the set of transmitter-receiver pairs of *simultaneous direct* transmissions active at a given time by $\mathcal{SD} := \{(S_1, D_1), (S_2, D_2), \ldots, (S_m, D_m)\}$. Also, we denote the set of transmitters by $\mathcal{S} := \{S_1, \ldots, S_m\}$. Note that these sets vary over time; if not otherwise indicated, however, we will consider one fixed but arbitrary time instant. For simplicity of notation, the node symbols are used also to represent their locations. For example, $|S_i - D_i|$ is the *Euclidean distance* between the nodes S_i and D_i in $\mathbb{R}^d$.

In both, the Protocol and the Physical Model the assigned transmission rate from node $S_i \in \mathcal{S}$ to node D_i is $W_i = W$ for a successful transmission where W is the *channel capacity*; for unsuccessful transmissions we set $W_i = 0$. Note that we assume a broadcast channel for wireless networks, so a transmission will typically be received by several nodes simultaneously. On one hand, broadcasting data to all neighbors can help to increase the throughput, on the other hand, simultaneous reception from different nodes is not feasible because of interference.

1) Protocol Model: Under the Protocol model a transmission is modeled as successful if $|S_k - D_i| \geq (1+\Delta)r$ for all $S_k \in \mathcal{S}\backslash\{S_i\}$, and $|S_i - D_i| \leq r$ where $\Delta > 0$ is the *interference parameter* and $r > 0$ the *transmission range*.

2) Physical Model: Under the Physical model a transmission is modeled as successful if

$$\mathrm{SINR} = \frac{PG_{ii}}{N_o + \sum_{k \neq i, k \in \mathcal{S}} PG_{ki}} \geq \beta \qquad (1)$$

Here, β is the SINR-threshold, N_o represents the ambient noise, and G_{ki} denotes the signal loss, meaning that PG_{ki} is the receiving power at node D_i from transmitter S_k. We assume a low power decay for the signal loss of the form $G_{ki} = |S_k - D_i|^{-\alpha}$, where $\alpha > d$ is the signal loss exponent.

We introduce the parameter $r_{\max}$ as the maximum possible distance between a transmitter and receiver to ensure a successful transmission under the Physical Model. Equation (1) implies that $r_{\max} = \left(\frac{P}{\beta N_o}\right)^{1/\alpha}$. Note that $r_{\max}$ is different than the radio range parameter r in Protocol Model. Here, a transmitter can achieve the range of $r_{\max}$ only if no other simultaneous transmission occurs in the network. We will use parameter $r_{\max}$ as transmission range (corresponding to the best case) to compute the minimum energy needed for transporting a given set of bits. However, for analyzing the maximum throughput, we apply (1) which models the effect interference on a set of simultaneous transmissions in the wireless network.

B. Connectivity Graph and Traffic Pattern

Wireless networks are usually modeled by *geometric graphs*. The nodes of network are the vertices V of the geometric graph. Two nodes are considered adjacent and connected by an edge $e \in E$, if the distance between them is less than a certain value r_G. We build the *connectivity graph* of a given wireless network by setting $r_G = r$ for the Protocol Model and $r_G = r_{\max}$ for the Physical Model in the geometric graph model. We represent the connectivity graph by $G = (V, E)$. It is easy to show that in order to establish some sessions in the wireless network there must exist a path in G between the source and the terminals of each session.

We represent multiple unicast sessions by the set of source-terminal pairs $\mathcal{AB} := \{(A_1, B_1), ..., (A_k, B_k)\}$ where $\mathcal{A} := \{A_1, \ldots, A_k\}$ and $\mathcal{B} := \{B_1, \ldots, B_k\}$ are the sets of sources and terminals. We refer to the *hop-count distance* of two nodes A_i from B_j by $\ell(A_i, B_j)$. For the networks with directed links, $\ell(A_i, B_j)$ is the hop-count length of shortest directed path from A_i to B_j. While the computation of the shortest paths in real-world scenarios are often hampered by incomplete information or other restrictions, using shortest paths is certainly appropriate when deriving theoretical bounds, as they constitute the best of worlds in terms of routing. Also, we define the distance of B_j from a set of nodes $\mathcal{A}$ as usual as $\ell(\mathcal{A}, B_j) = \min\{\ell(A_i, B_j) : A_i \in \mathcal{A}\}$.

C. Transport Capacity

The *transport capacity* is an important parameter of a wireless networks, which reflects the maximum sum of the rates and the number of transmissions of unicast sessions. Note that the unit of transport capacity is "bit-meter per second" which is different from the unit of throughput (bit per second). Interestingly, by computing the transport capacity of wireless network one can estimate the average throughput rate of the unicast sessions. The transport capacity of a set of source-terminal pairs $\mathcal{AB}$, is defined as:

$$C_T(\mathcal{AB}) := \max_{\text{multi-hop paths}} \sum_k |A_k - B_k| R_k \tag{2}$$

where R_k is the average rate of unicast session between of A_k and B_k over a given multi-hop path. The maximum is taken over all possible multi-hop routes establishing the required connections between the sources and terminals. A simple upper bound which actually does not depend on the set $\mathcal{AB}$ is found by noting that for the simultaneous routes achieving C_T there must be a time instance where the simultaneous direct hop-forwarding transmission reach at least C_T [1]. Therefore,

$$C_T(\mathcal{AB}) \le \max_{\mathcal{SD}} \sum_{(S_i, D_i) \in \mathcal{SD}} |S_i - D_i| W_i \tag{3}$$

where the maximum is over all possible sets of simultaneous, direct transmissions $\mathcal{SD}$, also W_i is the transmission rate of (S_i, D_i) transmitter-receiver pair. The transmission rate is computed based on the channel model.

D. Data Stream and Energy Consumption Model

We will use in our arguments a simple non-coding scheme which routes as commodity flows (replication, forwarding), a scheme we call the *flow scheme*. In contrast, we denote by a *coding scheme* any scheme using all of the operations of a flow scheme and in addition allowing the packets to be decoded or recoded at each node where they are received. In addition, in a coding scheme, intermediate nodes can send the results obtained from applying arbitrary functions to all previously received data and their own source data such with the only restriction that each destination node is able to decode the data intended for it from all of its received bits and local data.

We call a single multicast session or multiple unicast sessions *optimally source coded*, if the data streams generated by the source nodes are in optimally compressed format and independent from each other. More precisely, let the random process $X(A_i)$ denote the binary data stream generated by a source node A_i. We assume that $X(A_i)$ is in some optimal data compression format. Short, $X(A_i)$ has the maximum integral entropy rate. In addition, we assume that the random processes $X(A_i)$ for different A_i are independent of each other, meaning that different sources generate independent information. Agreeably, this manifests a simplifying yet appropriate assumption. Indeed, assuming some dependence among the generated information leads naturally to Distributed Source Coding (DSC) techniques for maximizing the throughput [25], [26]. However, the focus of this paper lies in pure *network coding gain* and not in (single nor multiple) source coding gain.

We quantize the *consumed energy* for transporting information by assuming that every transmission sends a constant amount of information, i.e. 1 bit. Consequently, the consumed energy for transporting the information becomes proportional to the *number of transmissions*. Note that we assume that each transmission consumes the same amount of energy, independently of distance.

As a particular consequence of our setting, we assume that the *header* is not used as information, so the only 1 bit of information that is transmitted is the content of each transmission. Consequently, we do not consider the overhead provided by routing, nor potential ways to exploit routing information. In addition, we assume that the information cannot be communicated between the nodes without transmission, in other words, timing or omission of transmissions does not provide any new information to the receiver nodes. This can give us a good estimation of energy consumption in real wireless networks when the packet size is large compare to the header size. Note that we also do not consider the energy consumption of the nodes used for carrier sensing, reception, MAC control packets in wireless communication.

From these assumptions, we can show that the expected number of transmissions for sending k specific bits of the data stream of some sources between two part of the network is at least k. Because the values of the bits are independent and each bit is equal to 0 or 1 with probability $1/2$ and also each transmission delivers 1 bit. Note that the expectation is computed over different realizations of data streams of the sources. In summary, the following assumptions are used for our analysis.

- A1 (Optimality source-coded data): Each source generates a data stream in an optimally compressed format.
- A2 (Independence of information for different sources): Different sources generate independent information.
- A3 (Energy consumption): The consumed energy for transporting the information is proportional to the number of transmissions and each transmission delivers 1 bit.

IV. Bounds on the Gain of Network Coding for Single Multicast

In this section we bound the benefit of network coding in terms of energy savings and throughput for a single multicast session in wireless networks.

A. Energy Gain of Network Coding for a Single Multicast

Here, we study the energy consumption of a single multicast session in an arbitrary wireless networks. Examples for wireless networks are provided in [20], [21] in which the benefit of network coding in terms of reducing the number of transmissions (energy saving) of a broadcast session achieve the factors of 2 and $4/3$, respectively. In [6], an example for a wired network with directed links is depicted where the gain of network coding on the throughput and energy is proportional to $\log(\#V)$, where $\#V$ is the number of nodes.

In Theorem 1 we prove that under assumption A1 and A3 the expected number of transmissions can be reduced by at most a constant factor for a single multicast session. Certainly, a coding scheme can reduce the number of transmissions of a multicast session. Nevertheless, as we will argue, the mutual location of source and terminals within the network topology together with the geometric properties of the wireless channel models enforce a minimal number of transmissions for transporting the bits under any arbitrary network coding. These minimal transmissions establish certain communication paths which can be exploited by a flow scheme to deliver the same information to the same terminals with just a constant factor more transmissions. As we show, this factor is surprisingly small.

The rigorous argument of Theorem 1 uses the following topological parameters. Let $\mathcal{M}$ be the set of all terminals together with the source of the single multicast session. Next, let $\mathcal{H}_0$ be a Maximal Independent Set (MIS) of $\mathcal{M}$, i.e. $\mathcal{H}_0$ is a set with maximal size such that no two nodes of $\mathcal{H}_0$ are neighbors in the connectivity graph $G = (V, E)$. By definition, we can show that every node of $\mathcal{M}$ is either in $\mathcal{H}_0$ or has at least one neighbor in $\mathcal{H}_0$. In wireless networks this means that every two nodes of $\mathcal{H}_0$ are in distance larger than radio range (r for the Protocol Model and $r_{\max}$ for the Physical Model) from each other. However, each node of $\mathcal{M}$ is located inside of the radio range of at least one node of $\mathcal{H}_0$.

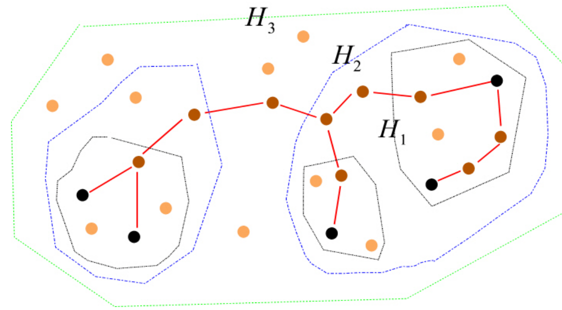

Fig. 1. The nodes of $\mathcal{M}$ have been colored black. The figure shows U which is built in 3 steps by considering the components of $\mathcal{H}_1$, $\mathcal{H}_2$, and $\mathcal{H}_3$.

Iteratively, let $\mathcal{H}_i = \{u \in V : \ell(u, \mathcal{H}_{i-1}) \leq 1\}$ for $i \geq 1$. Clearly, $\mathcal{H}_0 \subseteq \mathcal{H}_1 \subseteq \mathcal{H}_2 \subseteq \ldots$ (see Fig. 1). We also define $\mathcal{G}_0 = \mathcal{H}_0$ and $\mathcal{G}_i = \mathcal{H}_i \setminus \mathcal{H}_{i-1}$.

The following notation will come in handy: Let κ_d be the maximum number of nodes that can be placed in the unit d-dimensional sphere, such that every two of them have a distance strictly larger than 1. We find quickly that

$$\kappa_d := \begin{cases} 2 & \text{if } d = 1 \\ 5 & \text{if } d = 2 \\ 13 & \text{if } d = 3 \end{cases} \tag{4}$$

Further let

$$k := \min\{j : \text{the nodes of } \mathcal{H}_j \text{ are connected}\} \tag{5}$$

$$n_i := \#\{\text{connectivity components of } \mathcal{H}_i\} \tag{6}$$

Clearly, $n_0 \geq n_1 \geq \ldots \geq n_k = 1$.

Next, we establish three lemmas which will be instrumental for the proof of Theorem 1.

Lemma 1: Denote the number of transmissions for sending a multicast bit under an arbitrary coding scheme by N. Then under assumptions A1 and A3,

$$\mathbb{E}[N] \geq n_0/\kappa_d \tag{7}$$

Proof of Lemma 1: We compute a lower bound using some geometric properties of wireless networks and the channel model. By the geometric property of κ_d we find that every transmission can cover at most κ_d nodes of $\mathcal{H}_0$.

Next, we prove (7) using a contradiction argument. If (7) is not satisfied, then there exists a terminal in $\mathcal{H}_0$ where the expected number of transmissions which it receives per multicast bit is less than 1. That node cannot decode all bits sent by the source correctly, because the source sends optimally compressed information. ■

Lemma 2: Denote the number of transmissions for sending a multicast bit under an arbitrary coding scheme by N. Then under assumptions A1 and A3,

$$\mathbb{E}[N] \geq n_1 + \ldots + n_k \tag{8}$$

Proof of Lemma 2: Here, we derive the lower bound by considering the graph topology of the network introduced above. We consider the cutset between $\mathcal{H}_{i-1}$ and $V \setminus \mathcal{H}_{i-1}$ (the set of edges between $\mathcal{G}_{i-1}$ and $\mathcal{G}_i$). We claim that the expected number transmissions which is needed in the cutset for disseminating a multicast bit through $\mathcal{H}_{i-1}$ components is at least n_i. Again, we prove the claim by a contradiction argument. If the claim is not true, then for at least one of the connectivity components of $\mathcal{H}_i$ the expected number of transmissions in the cutset per multicast bit is less than 1. So, the terminals of that component cannot decode all generated bits correctly.

Note that we consider the direction of the transmission for a component of $\mathcal{H}_{i-1}$ from $\mathcal{G}_{i-1}$ to $\mathcal{G}_i$ if the component contains the source node, and from $\mathcal{G}_i$ to $\mathcal{G}_{i-1}$ if it does not contain the source. Since, $\mathcal{G}_0, \mathcal{G}_1, \ldots, \mathcal{G}_k$ represent a partitioning over the set of nodes. Therefore, the expected number of transmissions for a multicast bit is at least $n_1 + \ldots + n_k$. ■

Lemma 3: There exists a flow scheme for the multicast session which transports every bit using N' transmissions, where

$$N' \leq 3n_0 + 2(n_1 + \ldots + n_k) - 2k - 1 \tag{9}$$

Proof of Lemma 3: For the proof it is enough to show that there exists a set of nodes $U \subseteq V$, such that the size of U is equal to N' and the induced graph by U is connected, contains the source node, and covers all terminals.

We Build U in $k+1$ steps. First, we set $U_0 = \mathcal{H}_0$. Clearly, U_0 covers all nodes of $\mathcal{M}$. For $1 \leq i \leq k$, we let U_i be the set obtained by adding the minimal number of nodes U_{i-1} in order to reduce the number of connectivity components from n_{i-1} to n_i. The set U_k is our target set U.

Next, we bound the number of nodes which are added to U_{i-1} at step i. Without loss of generality, we may assume that $n_{i-1} > n_i$. This means $n_{i-1} - n_i$ connections are created among the components of $\mathcal{H}_{i-1}$ after building $\mathcal{H}_i$. We show that at most $2i(n_{i-1} - n_i)$ nodes are need to create those connections among the corresponding components.

As an illustrative example, consider $\mathcal{H}_{i-1}^{(1)}$ and $\mathcal{H}_{i-1}^{(2)}$ as two components of $\mathcal{H}_{i-1}$ which get connected after building $\mathcal{H}_i$. It follows from the definition of $\mathcal{H}_i$ that there exists a path of length $2i$ or $2i+1$ between two nodes of $\mathcal{H}_0$ which belong to $\mathcal{H}_{i-1}^{(1)}$ and $\mathcal{H}_{i-1}^{(2)}$. Because, from the definition, the nodes of $\mathcal{H}_i$ are at most i hop away from at least one node in $\mathcal{H}_0$. So, two components are connected as we build $\mathcal{H}_i$ from $\mathcal{H}_{i-1}$ only if one of the depicted scenarios in Fig. 2 occurs. Then, we add $2i-1$ or $2i$ nodes to the path which connect the terminals to U. This process is repeated $n_{i-1} - n_i$ times to connect all the corresponding components of $\mathcal{H}_{i-1}$.

So, by continuing this algorithm and increasing i, the set U_k gets connected in the k^{th} step. Finally, we add the source node to U_k if it does not belong to it already. Clearly, the size of U is bounded by $1 + n_0 + \sum_{i=1}^{k} 2i(n_{i-1} - n_i) = 3n_0 + 2(n_1 + n_2 + \ldots + n_{k-1} + n_k) - 2k - 1$. ■

Theorem 1: Consider a single multicast session which is optimally source coded in an arbitrary wireless network. The gain of network coding in terms of reducing the expected number of transmissions is less than a factor of $3\kappa_d + 2$, where κ_d is defined in (4).

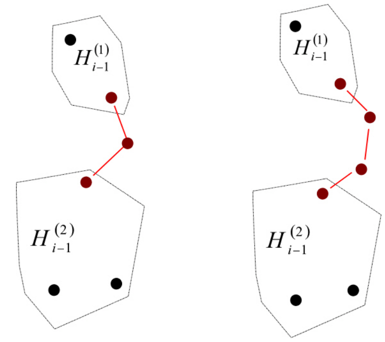

Fig. 2. Two components $\mathcal{H}_{i-1}^{(1)}$ and $\mathcal{H}_{i-1}^{(2)}$ are connected after building $\mathcal{H}_i$ if and only if one of these two scenarios occurs.

Proof of Theorem 1: Lemmas 1 and 2 give us lower bounds on the expected number of transmissions ($\mathbb{E}[N]$) under any arbitrary network coding scheme. Also, Lemma 3 shows that there exists a flow scheme which uses N' transmissions per multicast bit. It follows that the benefit of network coding in terms of reducing the expected number of transmissions is bounded by $N'/\mathbb{E}[N]$. From (7), (8) and (9), it easy to show that $N'/\mathbb{E}[N] < 3\kappa_d + 2$. ■

Remark: The proof of Theorem 1 also shows that the presented algorithm for building U in Lemma 3 can estimate the minimum multicast tree of a multicast session (Steiner tree) in a geometric graph up to the constant factor $3\kappa_d + 2$. To the best of our knowledge this, per se, constitutes a novel result in graph theory.

Next, we point out two particularly interesting scenarios, where concrete and simple bounds on the network coding gain can be given (see corollary 1). In the first case, the *long path scenario*, the source and the terminals are located far from each other (i.e. $\gamma_1 << 1$). Then, the gain of network coding is bounded by a constant close to 2. The intuitive reason behind this result lies in the fact that any network coding scheme still needs to establish some disjoint paths to the terminals which are located far from each other. In the second case, explicit knowledge of the multicast tree leads to tighter bounds on the network coding gain.

Corollary 1: Assume that $n_0, n_1, \ldots, n_k$ are defined as (6) for an optimally source coded single multicast session. Then the gain of network coding on the expected number of transmissions is bounded as the following

(i) If $n_0 = \gamma_1(n_1 + \ldots + n_k)$, the gain is bounded by a factor $3\gamma_1 + 2$ for $\gamma_1 < \kappa_d$ and $\kappa_d(3 + 2/\gamma_1)$ for $\gamma_1 > \kappa_d$.

(ii) If there exists a multicast tree with size of N_m such that $N_m \leq \gamma_2 \max(n_0/\kappa_d, n_1 + \ldots + n_k)$, the gain is bounded by a factor γ_2.

Proof of Corollary 1:

(i) The bound on $N'/\mathbb{E}[N]$ can be easily computed using Lemmas 1, 2, and 3.

(ii) Obviously, we have $N_m/\mathbb{E}[N] \leq \gamma_2$ using Lemmas 1 and 2. ■

B. Throughput Gain of Network Coding for a Single Multicast

The throughput of a single multicast sessions on a directed graph has been a popular case of study in network coding papers [6]–[9]. Here we prove that network coding can increase the throughput of a single multicast session in wireless networks by at most a constant factor, in contrast to wired network where the throughput can be increased unboundedly large using network codes [6].

We emphasize that the results of [16], [17] which are obtained for wired network with bidirectional are not applicable for wireless network model. There are two main differences between the wired and wireless network models which make the network coding gain very different in these networks.

First, in the wireless networks we assume that a node receives one signal at a time (simple point-to-point coding for wireless communication). So, the receiving rate of every node is bounded by the wireless channel capacity. However, in the wired network model a node can receive several different bits over several links simultaneously, and there is no such bound for the receiving rate.

Second, in wireless network model, noise and interference constitute additional limiting factors for the network capacity. The SINR model for noise and interference which form a part of one of our channel models lead to geometric constraints which in turn imply the fundamental bounds we derive. However, the wired network model is not limited by such geometric constraints and can be in the form of any arbitrary directed or undirected graph.

Theorem 2 shows that network coding can increase the throughput of a single multicast session by at most a constant factor.

Theorem 2: Assume Protocol or Physical Model for the wireless channel. Consider a single multicast session which is optimally source coded in an arbitrary wireless network. Then, the network coding gain on the throughput of the multicast session is at most a constant factor c, where

$$c := \begin{cases} \lceil 2 + (1+\Delta)\sqrt{d+3}\rceil^d \\ (5^d - 2^d)\left\lceil \sqrt{d}\left(2 + \left(\frac{\beta \sum_{J\in\mathbb{Z}^d}^{|J|>0} |J|^{-\alpha}}{1-\rho^{-\alpha}}\right)^{\frac{1}{\alpha}}\right)\right\rceil^d \end{cases} \tag{10}$$

under Protocol Model and Physical Model respectively ($\rho > 1$ is a constant).

Proof of Theorem 2: Consider an arbitrary terminal of the multicast session. The maximum rate at which data can be received by the terminal is equal to the wireless channel capacity W. By considering the cutset which separates the terminal from other nodes, we conclude that W is an upper bound on the throughput of a multicast session under any arbitrary network coding.

On the other hand, in [27], [28] flow scheme for Protocol Model and Physical Model are constructed with a broadcast throughput of W/c for any connected wireless network, where c is some constant determined by the channel model as (10). Such flow schemes can be used to send the data from the source to all terminals with rate W/c. This shows that network coding can improve the throughput of a single multicast session by at most a factor c. ■

Note that despite its apparent similarity to the proofs of previous work [27], [28], our argument is in fact different since network coding has not been taken into account in the mentioned existing work. Here, we study the gain of network coding on the throughput a multicast session.

V. Bounds on the Gain of Network Coding for Multiple Unicast

In this section we study the benefit of network coding for multiple unicast sessions in terms of energy and throughput. By assumption, energy savings are proportional to savings in the number of transmissions needed. We first bound the gain of network coding in terms of the number of transmissions, then in terms of the throughput.

A. Energy Gain of Network Coding for Multiple Unicasts

Here, we investigate the benefit of network coding in terms of energy saving for multiple unicast sessions. Note that in the single multicast scenario, network coding benefits by distributing the same information on different links and by efficiently using the links for sending the information toward several terminals. However, in multiple unicasts scenario, the links carry some independent information of different unicast sessions (assumption A2). Intuitively, there is less chance for network coding to benefit in terms of the number of transmissions for unicast sessions. We will explore this fact more precisely later.

Theorem 3 provides a lower bound in terms of the expected number of transmissions for multiple unicast sessions. The lower bound is determined simply by considering the hop-count distance of the source and terminal nodes. Interestingly, the theorem is also valid for wired networks under assumptions A1, A2, and A3.

Notably, the bounds of Theorem 3 apply to any set of optimally coded bits with independent information sent in unicast from a set of senders to their terminals. If the bits considered are such that the set of sources and the set of terminals are distant from each other, the lower bound becomes tighter.

Theorem 3: Assume that $b_1, b_2, \ldots, b_k$ are optimally source coded bits of some simultaneous unicast sessions. Denote N as the number of transmissions used by an arbitrary coding scheme for transporting these bits. Also, denote the source and terminal of bit b_j by A_j and B_j for $j = 1, \ldots, k$ (a node can be the source or destination for several bits). Then,

$$\mathbb{E}[N] \geq \max[\sum_{j=1}^{k} \ell(A_j, \mathcal{B}), \sum_{j=1}^{k} \ell(\mathcal{A}, B_j)] \tag{11}$$

Remark: We should point out that the source and the terminal of a unicast session are repeated equal to the number of bits of that the session has among $b_1, b_2, \ldots, b_k$ in the sum of equation (11). When the average rate of unicast sessions is given, we can choose the number of bits of each session

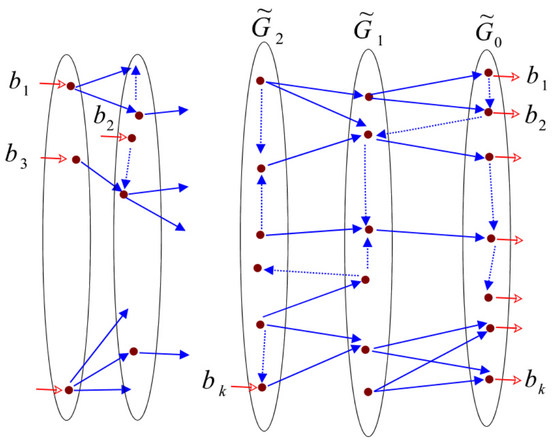

Fig. 3. Grouping the nodes in terms of their distances from the set of terminals.

proportional to its average rate in order to compute a lower bound for the rate of energy consumption.

Proof of Theorem 3: We group the nodes of the network in terms of their distances from the set of terminals ($\mathcal{B}$). We define $\tilde{\mathcal{H}}_0 = \mathcal{B}$ and $\tilde{\mathcal{H}}_i = \{u \in V : \ell(u, \mathcal{B}) \leq i\}$. Also, we define $\tilde{\mathcal{G}}_0 = \tilde{\mathcal{H}}_0$ and $\tilde{\mathcal{G}}_i = \tilde{\mathcal{H}}_i \setminus \tilde{\mathcal{H}}_{i-1}$. It is easy to show that the nodes of $\tilde{\mathcal{G}}_i$ are only connected to the nodes of $\tilde{\mathcal{G}}_{i-1}$, $\tilde{\mathcal{G}}_i$ and $\tilde{\mathcal{G}}_{i+1}$. Fig. 3 shows the transmissions which carry the bits among different groups.

Denote the number of sources which are not in $\tilde{\mathcal{H}}_i$ by q_i. Clearly, $q_i = \sum_j \mathbb{I}_{[\ell(A_j,\mathcal{B})>i]}$. Note that the cutset between $V \setminus \tilde{\mathcal{H}}_i$ to $\tilde{\mathcal{H}}_i$ is the set of edges between $\tilde{\mathcal{G}}_{i+1}$ and $\tilde{\mathcal{G}}_i$ (in directed graph, consider the edges directed from $\tilde{\mathcal{G}}_{i+1}$ to $\tilde{\mathcal{G}}_i$).

From the assumptions $b_1, b_2, \ldots, b_k$ are independent optimally compressed bits. So, for transmitting them from $V \setminus \tilde{\mathcal{H}}_i$ to $\tilde{\mathcal{H}}_i$ the expected number of transmissions is at least q_i.

Therefore, the expected number of transmissions for transporting the bits all unicast sessions under any arbitrary network coding is at least $\sum_{i=0}^{\infty} q_i$. Then we have

$$\begin{aligned}\mathbb{E}[N] &\geq \sum_{i=0}^{\infty} q_i = \sum_{i=0}^{\infty}\sum_{j=1}^{k} \mathbb{I}_{[\ell(A_j,\mathcal{B})>i]} \\ &= \sum_{j=1}^{k}\sum_{i=0}^{\infty} \mathbb{I}_{[\ell(A_j,\mathcal{B})>i]} = \sum_{j=1}^{k} \ell(A_j, \mathcal{B})\end{aligned}$$

Similarly, by grouping the nodes in terms of distance from the set of sources ($\mathcal{A}$) and applying the same method, we can show that $\mathbb{E}[N] \geq \sum_{j=1}^{k} \ell(\mathcal{A}, B_j)$. ■

Two symmetrical cases of particular interest lead to a simple yet striking conclusion of when network coding should not be employed. Namely the case when independent unicast connections have *either the same source or the same terminal*. Surprisingly, network coding provides *no benefit* at all in terms of the number of transmissions in these cases (see Corollary 2).

Examples of such scenarios include *sensor networks* where the sensors send independent sensing information to a set of sinks. Note that every sensor needs to send its information to at least one of the sinks, but it does not matter which one. If we consider a virtual node (say a super-sink) which receives the information of the sinks by direct (wired) links, then the traffic becomes similar to case (i) of Corollary 2 below.

Another concrete example concerns a certain part of traffic in mesh networks with an *internet gateway*, namely independent uplink traffic toward gateway (case (i)) and independent downlink traffic from the gateway (case (ii)). So, in mesh networks, network coding does not benefit the energy consumption for uplink and downlink traffic separately. However, note that network coding would still benefit by combining the uplink and downlink packets [14], [15] or for transporting multicast and broadcast packets [20], [21].

Note that [26] studies case (i) more generally by considering a cost function in a single sink sensor networks. The paper shows that when the sources are independent and the cost is proportional to the number of transmissions, then shortest path routing protocol has the minimum cost. This is agrees with our result which is proved in by a different technique.

Corollary 2: Consider multiple unicast sessions with optimally source coded data stream. For the following two scenarios, there is no network coding benefit in terms of the number of transmissions (energy).

(i) A set of sources send their independent information to a single sink.

(ii) A single source sends independent information to each of a set of terminals.

Proof of Corollary 2: We establish the claim by showing that the shortest-path flow scheme is the most efficient scheme among all flow and coding schemes. If we use the flow scheme which transports the bits over the shortest path, then the number of transmissions is equal to $\sum_{i=1}^{k} \ell(A_i, \mathcal{B})$ in case (i) and equal to $\sum_{i=1}^{k} \ell(\mathcal{A}, B_i)$ in case (ii) of Theorem 3. So, we conclude that network coding does not benefit in terms of reducing the number of transmissions for these scenarios. ■

B. Throughput Gain of Network Coding in Wireless Networks

Here, we bound the gain of network coding on the throughput of multiple unicast sessions in wireless networks. We study the transport capacity and the throughput of the sessions under an arbitrary coding scheme.

Theorem 4 shows that the maximum transport capacity of an arbitrary network under an arbitrary coding scheme is bounded by constant factor π of the maximum transport capacity computed under the flow scheme. Therefore, the gain of network coding on the maximum transport capacity of arbitrary wireless network is bounded by a factor of π.

Moreover, the theorem shows that the computed upper bounds on the transport capacity in previous works [1]–[4] increase by at most factor of π when we employ network coding in an arbitrary wireless network.

Theorem 4: Denote the maximum transport capacity of an arbitrary wireless network by using the flow scheme and coding scheme by C_T^{f} and C_T^{nc} respectively. Then,

$$C_T^{\mathrm{nc}} \leq K_d \cdot C_T^{\mathrm{f}} \tag{12}$$

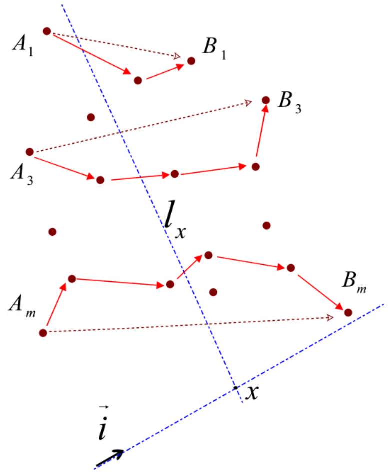

Fig. 4. l_x is an geometric cutset for some source-terminal pairs of $\underline{\mathcal{AB}}$.

where $K_d = 2$ if $d = 1$ and $K_d = \pi$ if $d = 2, 3$ (d is the dimension of the space).

Proof of Theorem 4: We prove the theorem in $d = 2$ dimensional space. The proof can be easily extended for $d = 1, 3$ dimensional space using the same method.

We use the following lemma for our arguments. The proof of lemma is in the technical report of this work [29].

Lemma 4: Consider an arbitrary set of vectors $Q = \{\vec{a_1}, \ldots, \vec{a_k}\}$ in d-dimensional space. Then, there exists an unit vector $\vec{i}$ and a set $\underline{Q} \subseteq Q$ such that

$$\sum_{\vec{a_j} \in Q} |\vec{a_j}| \leq K_d \sum_{\vec{a_j} \in \underline{Q}} \vec{i} \cdot \vec{a_j} \tag{13}$$

(K_d is the same as Theorem 4).

For the proof of theorem, we consider an arbitrary set of unicast sessions $\mathcal{AB} = \{(A_1, B_1), \ldots, (A_k, B_k)\}$ with average rates $R_1, \ldots, R_k$. Next, we define $\vec{a_j} = R_j \cdot \overrightarrow{A_jB_j}$. By Lemma 4, there exists an unit vector $\vec{i}$ and $\underline{\mathcal{AB}} \subseteq \mathcal{AB}$ such that

$$\sum_{(A_j,B_j) \in \mathcal{AB}} R_j |\overrightarrow{A_jB_j}| \leq K_d \sum_{(A_j,B_j) \in \underline{\mathcal{AB}}} R_j \overrightarrow{A_jB_j} \cdot \vec{i} \tag{14}$$

Then, we rotate the Cartesian axes such that the axis X is aligned on the direction of unit vector $\vec{i}$. We denote the orthogonal line which crosses axis X at point x by l_x (see Fig. 4). Also, we denote $\mathbb{I}_{[A_jB_j \leftrightsquigarrow l_x]}$ as the indicator function that the line segment A_jB_j has been intersected by line l_x. Note that if $(A_j, B_j) \in \underline{\mathcal{AB}}$ and $\mathbb{I}_{[A_jB_j \leftrightsquigarrow l_x]} = 1$, then we can show that A_j is located in the left side and B_j is located in the right side of l_x because $\overrightarrow{A_jB_j} \cdot \vec{i} > 0$ (see the proof of Lemma 4). Therefore, $\sum_{(A_j,B_j) \in \underline{\mathcal{AB}}} R_j \mathbb{I}_{[A_jB_j \leftrightsquigarrow l_x]}$ is the average rate of information for the unicast sessions of $\underline{\mathcal{AB}}$ which go from the left to the right side of line l_x .

Denote the set of simultaneous transmissions at time instant τ by $\mathcal{SD}_\tau = \{(S_1, D_1), \ldots, (S_m, D_m)\}$. Then, the rate of bits are transmitted across l_x at this time instant is $\sum_{(S_j,D_j) \in \mathcal{SD}_\tau} W_j \mathbb{I}_{[S_jD_j \leftrightsquigarrow l_x]}$. Therefore, from the definitions we have

$$\sum_{(A_j,B_j) \in \underline{\mathcal{AB}}} R_j \mathbb{I}_{[A_jB_j \leftrightsquigarrow l_x]} \leq \lim_{T \to \infty} \frac{1}{T} \int_0^T \sum_{(S_j,D_j) \in \mathcal{SD}_\tau} W_j \mathbb{I}_{[S_jD_j \leftrightsquigarrow l_x]} d\tau \tag{15}$$

Now, we take integral by moving l_x from $x = -\infty$ to ∞,

$$\begin{aligned}
&\sum_{(A_j,B_j) \in \underline{\mathcal{AB}}} R_j \overrightarrow{A_jB_j} \cdot \vec{i} \\
&= \sum_{(A_j,B_j) \in \underline{\mathcal{AB}}} R_j \int_{-\infty}^{\infty} \mathbb{I}_{[(A_j,B_j) \leftrightsquigarrow l_x]} \, dx \\
&= \int_{-\infty}^{\infty} \sum_{(A_j,B_j) \in \underline{\mathcal{AB}}} R_j \mathbb{I}_{[(A_j,B_j) \leftrightsquigarrow l_x]} \, dx \\
&\leq \int_{-\infty}^{\infty} \lim_{T \to \infty} \frac{1}{T} \int_0^T \sum_{(S_j,D_j) \in \mathcal{SD}_\tau} W_j \mathbb{I}_{[S_jD_j \leftrightsquigarrow l_x]} d\tau \, dx \\
&= \lim_{T \to \infty} \frac{1}{T} \int_0^T \sum_{(S_j,D_j) \in \mathcal{SD}_\tau} W_j \int_{-\infty}^{\infty} \mathbb{I}_{[S_jD_j \leftrightsquigarrow l_x]} dx \, d\tau \\
&= \lim_{T \to \infty} \frac{1}{T} \int_0^T \sum_{(S_j,D_j) \in \mathcal{SD}_\tau} W_j |\overrightarrow{S_jD_j} \cdot \vec{i}| \, d\tau \\
&\leq \lim_{T \to \infty} \frac{1}{T} \int_0^T \sum_{(S_j,D_j) \in \mathcal{SD}_\tau} W_j |\overrightarrow{S_jD_j}| \, d\tau \\
&\leq \max_{\mathcal{SD},\tau} \sum_{(S_j,D_j) \in \mathcal{SD}_\tau} W_j |\overrightarrow{S_jD_j}| = C_T^{\mathrm{f}}
\end{aligned}$$

Finally, by (14) we conclude that

$$C_T^{\mathrm{nc}} = \max_{\mathcal{AB}} \Big(\sum_{(A_j,B_j) \in \mathcal{AB}} R_j |\overrightarrow{A_jB_j}| \Big) \leq K_d \cdot C_T^{\mathrm{f}} \tag{16}$$

■

Next, we study the gain of network coding on the throughput of large homogeneous networks which is a popular model in network capacity papers [1]–[3], [30], [31]. Corollary 3 proves that network coding does not change the asymptotic behavior of the throughput of large homogeneous wireless networks.

Prior work (see [5]) shows a similar result under the Protocol Model for the wireless channel. Here, we establish the bound also under other channel models, i.e., the non-symmetric Protocol Model [1], [2], the Physical Model, and the Generalized Physical Models [3]. Further, our technique of proof is definitely different from [5] since we base our argument on the transport capacity.

Corollary 3: Network coding gain on the throughput of large homogeneous wireless networks is bounded by a constant factor.

Proof of Corollary 3: In previous works [1]–[3] provide some asymptotic upper bounds on the throughput (sum of the rates of unicast sessions) of large homogeneous networks under various channel models. They do so in two steps. First, an upper bound on the transport capacity (C_T^{f}) is computed. Second, it is divided by the average distance of source-terminal pairs (L). On the other hand, [1], [2], [30], [31] provide some flow schemes to achieve a throughput within a constant factor of the computed upper bounds. The throughput of these schemes represent tight lower bounds on the throughput capacity of the network.

Now, by Theorem 4, the gain of network coding on the transport capacity is bounded by a factor of K_d. Therefore, if we employ network coding then the throughput will be smaller than K_d multiply by the traditional upper bounds [1]–[3] (which have been computed for flow schemes). We also note the flow schemes can achieve the traditional upper bounds up to a constant factor [1], [2], [30], [31]. From these, we conclude that the network coding gain on the throughput is bounded by a constant. ■

VI. Conclusion

In this work we studied fundamental limitations of the benefit of network coding for arbitrary wireless multihop networks. We focused on two popular network scenarios: single multicast session and multiple unicast sessions. We proved the benefit of network coding in terms of throughput or energy saving is bounded by a constant factor for a single multicast session. Also, we computed bounds for the gain of network coding in terms of number of transmissions for multiple unicast sessions. Interestingly, we showed that network coding has no benefit in terms of energy in sensor networks where the sensors gather independent information for the sink or in mesh networks for unidirectional traffic from/towards the gateway. In addition, we proved that network coding can increase the transport capacity of an arbitrary wireless network by at most a factor of π. This result verifies that network coding does not change the throughput of large homogeneous networks more than a constant. The established bounds, complemented with previous related work on capacity bounds [1]–[5], seem to indicate that network coding plays valuable but not crucial role with regards to the performance of wireless networks. Rather, channel interference and topology of wireless networks seem to emerge as the determinant parameters on the throughput and energy consumption.

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