Shot-Noise Cascades

Rudolf Riedi
Rice Univ., Houston Texas / EIF-FR & FFHS Switzerland

Scaling and related phenomena form a striking and crucial component directly impact performance in a variety of applications, notably in networking and finance. We study the *Cascade of Pulses* $\{Q_r(t)\}_t$

$$Q_r(t) := \prod_{R_i > r} \pi\left(W_i, \tfrac{t-T_i}{R_i}\right). \tag{1}$$

where $(T_i, R_i; W_i)$ form a marked Poisson Point Process on $\mathbb{R} \times (0,1]$ with density $dm(t,a)$, a setting which allows for infinitely divisible scaling, strict stationarity and for deviations from powerlaws. Examples of interest include

$$\pi(w,t) := \begin{cases} 1 + (w-1)h(t) & \text{multiplicative shot-noise pulse,} \\ w^{k(t)} & \text{exponential shot-noise pulse.} \end{cases} \tag{2}$$

which reduce to the well-known cylindrical product of pulses when h or k is the indicator of $[-1/2, 1/2[$; also, the exponential shot-noise cascade can be written as an infinitely divisible cascade over a Poisson Counting Measure.

To the best of our knowledge, this is the first analysis of the cascade with pulses that are *not compactly supported*. The pulses having potentially infinite support precludes the use of certain standard techniques for establishing its multifractal properties. As a first step towards overcoming this hurdle, we establish moment and regularity conditions under which an appropriate limiting cascade $A(t) = \lim_{r\to 0} \int_0^t \frac{Q_r(s)}{\mathbb{E}[Q_r(s)]} ds$ can be defined and exhibits the scaling

$$\mathbb{E}|A(t+\delta) - A(t)|^q \simeq \delta^q \cdot \frac{\mathbb{E}[Q_r(t)^q]}{\mathbb{E}[Q_r(t)]^q} = \delta^{q - c(\rho(q) - q\rho(1))}$$

where $\rho(q) = \mathbb{E}[\int_{-\infty}^{\infty} \pi(W,u)^q - 1du]$ and $dm(t,a) = c/a^2 dtda$. Conditions and formulas take simple forms in the shot-noise cases. Financial support in part by the Fernfachhochschule Schweiz and by NSF grant ANI-0338856.